MTH 234 Exam 1 Name ____________________________

June 2, 2016 (75 points total)

Consider the vectors $\vec{u} = \langle 2, -1, 1 \rangle$, $\vec{v} = \langle -1, 1, 0 \rangle$, $\vec{w} = \langle 7, -2, 5 \rangle$, and the point $P_0 = (1, -3, 2)$

1. (4 points) Compute the distance from P_0 to the plane containing the origin and the vectors $\vec{u}$ and $\vec{v}$.

2. (4 points) Do the vectors $\vec{u}, \vec{v}$, and $\vec{w}$ all lie in the same plane? Justify your response.

3. (4 points) Compute the equation of the line containing P_0 which is orthogonal to both $\vec{u}$ and $\vec{v}$

4. (5 points) Find a vector function which represents the intersection of the surfaces:

$$y^2 + z^2 = 1 \quad \text{and} \quad x + z = 2.$$

5. (7 points) Find the equation of the line given by the intersection of the planes

$$2x + y + 8z = 3 \quad \text{and} \quad x + y + 5z = 2.$$

6. (8 points) A particle moves with position $\vec{r} = \left\langle t^2/2, t, \ln(t+1) \right\rangle$. Compute the normal and tangential components of acceleration.

7. (5 points) Compute the equation of the plane tangent to the surface $4x^2y + y^2 = z^2 + 1$ at the point (1,1,2).

8. (4 points each) Do the following limits exist? If so, give the value; if not, explain why the limit does not exist.

(a)

$$\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$$

(b)

$$\lim_{(x,y)\to(0,0)} \frac{x^2y^2e^y}{x^4-4y^2x^2}$$

9. (10 points) Suppose you are given a function $f(x, y, z) = x(y+1)^2 + x^3 z$ and the relations $x(s, t) = st$, $y(s, t) = s - t^2$, and $z(s, t) = t$. Compute the partial derivatives

$$\left(\frac{\partial f}{\partial s}\right)_t \quad \text{and} \quad \left(\frac{\partial f}{\partial t}\right)_s .$$

10. (10 points) Compute the directional derivative $D_{\hat{u}} f$ of $f(x, y, z) = xe^y \sin z$ in the direction $\hat{u} = \langle a, b, c \rangle$ at the point $(1, \ln 2, \pi/4)$

11. (5 points each) Consider the relation $xy^2 + y = ze^x$ where $x = se^t$ and $y = st$.

(a) Compute $\partial z/\partial t$ at $(s, t) = (3, 2)$
(b) Compute $\partial x/\partial y$ at $(s, t) = (3, 2)$