Math 2401 (K1-K3)
2/9/2015

Worksheet 7

Partial Derivatives, Chain Rule, Directional Derivatives

1. For each of the functions below, find all the first order partial derivatives:

a). $f(x, y) = xy^3 + x^2y^2$.

b). $f(x, y) = xe^{2x+3y}$.

c). $f(x, y) = \frac{x-y}{x+y}$.

d). $f(x, y) = 2x\sin(x^2y)$.

e). $f(x, y, z) = x\cos z + x^2y^3e^z$.

2. Show that the function $u(x, y) = \ln(1 + xy^2)$ satisfies the partial differential equation:

$$2\frac{\partial^2 u}{\partial x^2} + y^3\frac{\partial^2 u}{\partial y \partial x}.$$

3. If $g(s, t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies the equation:

$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0.$$

4. A function f is said to be **homogeneous of degree** n if it satisfies the equation:

$$f(tx, ty) = t^n f(x, y) \quad (1)$$

for all real t, where n is a positive integer and f has continuous second order partial derivatives.

a). Verify that the function $f(x, y) = x^2y + 2xy^2 + 5y^3$ is homogeneous of degree 3.

b). Show that if f is homogeneous of degree n, then f satisfies:

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y). \quad (2)$$

Hint: use the Chain Rule to differentiate both sides of (1) with respect to t, then give t an appropriate value in order to obtain (2).

5. Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin(xy)$ at the point $(1, 0)$ has the value 1.

6. Let f be a function of two variables that has continuous partial derivatives, and consider the points:

$$A(1, 3);\ B(3, 3);\ C(1, 7);\ D(6, 15).$$

The directional derivative of f at A in the direction of the vector $\overrightarrow{AB}$ is equal to 3, and the directional derivative at A in the direction of the vector $\overrightarrow{AC}$ is equal to 26. Find the directional derivative of f at A in the direction of the vector $\overrightarrow{AD}$.