Worksheet 6

1. Find and sketch the domain of each of the functions:

a). $f(x,y) = \sqrt{y - x - 2}$

b). $f(x,y) = \arcsin(y - x)$

c). $f(x,y) = \frac{1}{\ln(4 - x^2 - y^2)}$

2. For each of the functions above, find their range.

3. Consider the function:
$$f(x,y) = \frac{xy}{x^6 + y^2}.$$

a). Find the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the line $y = x$.

b). Find the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the line $y = 2x$. What can you conclude from these results?

c). You can do this faster by exploring the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along linear paths all at once. Namely, find the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along $y = kx$ for some real number k and draw the same conclusion.

4. Consider the function:
$$f(x,y) = \frac{x^3 y}{x^6 + y^2}.$$

Show that the limit of $f(x,y)$ as $(x,y) \to (0,0)$ does not exist by exploring the paths $y = kx$, $y = kx^2$, and $y = kx^3$.