Math 2401 (K1-K3)
1/14/2015

Worksheet 4 - Intersecting Planes

- The angle between two planes in space is defined to be the angle between their normal vectors.
- Two planes are parallel if and only if their normal vectors are parallel.
- Two non-parallel planes intersect in a line.

Example: Consider the planes:

$$3x - 6y - 2z = 15, \qquad \vec{n_1} = \langle 3, -6, -2 \rangle$$

and:

$$2x + y - 2z = 5. \qquad \vec{n_2} = \langle 2, 1, -2 \rangle$$

a). Find the angle between the planes.

b). Are the two planes parallel? If not, find parametric equations for the line of intersection of the two planes.

a). $\cos\theta = \dfrac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|} = \dfrac{6-6+4}{\sqrt{49}\sqrt{9}} = \dfrac{4}{21} \Rightarrow \boxed{\theta = \cos^{-1}(4/21)}$

b). The planes are not parallel, because their normal vectors are not scalar multiples of each other.

Line of Intersection L :

Parallel Vector : The line L is perpendicular to both $\vec{n_1}$ & $\vec{n_2}$, so a vector parallel to the line is $\vec{v} = \vec{n_1} \times \vec{n_2}$:

$$\vec{v} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = \langle 14, 2, 15 \rangle$$

Point on L : Just find any point that belongs to both planes. For example, take $z=0$ and solve for x & y :

$$\begin{cases} 3x - 6y = 15 \\ 2x + y = 5 \end{cases} \quad \begin{cases} x - 2y = 5 \\ 2x + y = 5 \end{cases} \; |\times 2 \quad \begin{array}{r} x - 2y = 5 \\ 4x + 2y = 10 \\ \hline \oplus \quad 5x = 15 \end{array} \Rightarrow x = 3 \Rightarrow y = -1$$

So a point on L is: $(3, -1, 0)$

Parametric Equations :

$$\begin{cases} x = 3 + 14t \\ y = -1 + 2t \\ z = 15t \end{cases}$$