Math 2401 (D1-D3)
8/25/2014

Worksheet 2 - Chapter 12

1. Find a vector with length 4 and direction $\frac{1}{\sqrt{3}}\vec{i} - \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}$.

$$\frac{4}{\sqrt{3}}\vec{i} - \frac{4}{\sqrt{3}}\vec{j} + \frac{4}{\sqrt{3}}\vec{k}$$

2. Find the distance between the points $P_1(4, 1, 2)$ and $P_2(4, -2, 6)$. Write the vector $\overrightarrow{P_1P_2}$ in component form. What is the length of the vector $\overrightarrow{P_1P_2}$? What is the direction of the vector $\overrightarrow{P_1P_2}$?

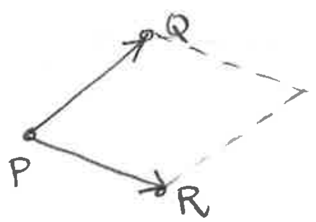
$$\text{dist}(P_1, P_2) = \sqrt{(4-4)^2 + (-2-1)^2 + (6-2)^2} = \sqrt{25} = \boxed{5}$$

$$\overrightarrow{P_1P_2} = -3\vec{j} + 4\vec{k} \;;\; |\overrightarrow{P_1P_2}| = 5 \;;\; \text{direction of } \overrightarrow{P_1P_2}: \; -\frac{3}{5}\vec{j} + \frac{4}{5}\vec{k}.$$

3. Given the vectors $\vec{u} = \vec{i} - \vec{j} + 2\vec{k}$ and $\vec{v} = 3\vec{i} - \vec{k}$, find $\cos\theta$, where θ is the angle between $\vec{u}$ and $\vec{v}$.

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{1\cdot 3 + (-1)\cdot 0 + 2(-1)}{\sqrt{6}\cdot\sqrt{10}} = \boxed{\frac{1}{\sqrt{60}}} = \boxed{\frac{1}{2\sqrt{15}}}$$

4. Find a unit vector orthogonal to the plane determined by the points $P(2, -2, 1)$ and $Q(-1, 0, -2)$ and $R(0, -1, 2)$. Find the area of the triangle ΔPQR.



$$\overrightarrow{PQ} = \langle -3, 2, -3\rangle \;;\; \overrightarrow{PR} = \langle -2, 1, 1\rangle$$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & -3 \\ -2 & 1 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} -3 & -3 \\ -2 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} -3 & 2 \\ -2 & 1 \end{vmatrix}$$

$$= 5\vec{i} + 9\vec{j} + \vec{k}$$

$$|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{25 + 81 + 1} = \sqrt{107}$$

Unit vector orthogonal to the plane: $\boxed{\frac{1}{\sqrt{107}}(5\vec{i} + 9\vec{j} + \vec{k})}$

Area of ΔPQR: $\boxed{\frac{\sqrt{107}}{2}}$

5. Find the ~~area~~ volume of the parallelepiped determined by the vectors $\vec{u} = 2\vec{i} - \vec{k}$, $\vec{v} = -2\vec{i} + \vec{j}$, and $\vec{w} = \vec{i} + 2\vec{j} - 2\vec{k}$.

$$A = \begin{vmatrix} 2 & 0 & -1 \\ -2 & 1 & 0 \\ 1 & 2 & -2 \end{vmatrix} = 2\begin{vmatrix} 1 & 0 \\ 2 & -2 \end{vmatrix} - 1\cdot\begin{vmatrix} -2 & 1 \\ 1 & 2 \end{vmatrix} = 2(-2) - (-4-1) = \boxed{1}$$