

Worksheet 2 - Vectors and the Dot Product

1. Classify the quantities below as either scalar or vector quantities:

- a). Velocity of a car.
- b). Speed of a car.
- c). Number of students at Georgia Tech.
- d). Mass of an orange.
- e). Weight of an orange.

2. Suppose that $\|\mathbf{u}\| = \sqrt{2}$ and $\|\mathbf{v}\| = 1$.

- a). Find $\mathbf{u} \cdot \mathbf{v}$, if the angle θ between \mathbf{u} and \mathbf{v} is $\theta = \pi/4$.
- b). Find $\mathbf{u} \cdot \mathbf{v}$, if the angle θ between \mathbf{u} and \mathbf{v} is $\theta = 3\pi/4$.

3. Prove that if:

$$\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2,$$

then \mathbf{u} and \mathbf{v} are orthogonal.

4. Let \mathbf{u} and \mathbf{v} be unit vectors, with \mathbf{u} making an angle of 45° with the positive x -axis, and \mathbf{v} making an angle of 60° with the positive x -axis. Use the dot product of these two vectors to compute $\cos(15^\circ)$.

5. The vector $\mathbf{b} = \langle 2, 2, 1 \rangle$ makes an angle of $\pi/6$ with the vector \mathbf{a} . Given that $\mathbf{a} \cdot \mathbf{b} = 6$, find $\|\mathbf{a}\|$.

6. Consider the points $P(a, 1, -1)$, $Q(0, 1, 1)$ and $R(a, -1, 3)$, for some real number a . Find a such that the angle $\angle PQR$ is a right angle.

7. Determine whether or not the pairs of vectors are orthogonal:

- a). $\mathbf{u} = \langle \cos \theta, \sin \theta, -1 \rangle$ and $\mathbf{v} = \langle \cos \theta, \sin \theta, 1 \rangle$.
- b). $\mathbf{u} = \langle \cos \theta, \sin \theta, -1 \rangle$ and $\mathbf{v} = \langle \sin \theta, -\cos \theta, 1 \rangle$.

8. Prove the Cauchy-Schwartz inequality:

$$|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\| \|\mathbf{v}\|.$$

9. Prove the Parallelogram Law:

$$\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2(\|\mathbf{u}\|^2 + \|\mathbf{v}\|^2).$$