

**Worksheet 17 - Surface Integrals**

1. Find  $\iint_S y \, d\sigma$ , where  $S$  is the surface  $z = x + y^2$ ,  $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$  (pictured in Figure 1). Try to use both methods (implicit and parametrized).
2. Find  $\iint_S (x^2 z + y^2 z) \, d\sigma$ , where  $S$  is the upper hemisphere:  $x^2 + y^2 + z^2 = 4$ ;  $z \geq 0$ .
3. Find the surface area of the portion of the sphere  $x^2 + y^2 + z^2 = 16$  that remains after removing its spherical caps that lie inside the cylinder  $x^2 + z^2 = 12$ .
4. Find  $\iint_S y \, d\sigma$ , where  $S$  is the portion of the cylinder  $x^2 + y^2 = 3$  between the planes  $z = 0$  and  $z = 6$ .
5. Find  $\iint_S x^2 y z \, d\sigma$ , where  $S$  is the portion of the plane  $z = 1 + 2x + 3y$  that lies above  $0 \leq x \leq 3$  and  $0 \leq y \leq 2$  - see Figure 2.
6. Find  $\iint_S (y + z) \, d\sigma$ , where  $S$  is the closed surface whose sides are given by the cylinder  $x^2 + y^2 = 3$ , whose top lies in the plane  $z = 4 - y$ , and whose bottom is in the  $xy$ -plane.
7. Find  $\iint_S y z \, d\sigma$ , where  $S$  is the surface parametrized by  $x = uv$ ,  $y = u + v$ ,  $z = u - v$ , where  $u^2 + v^2 \leq 1$ .

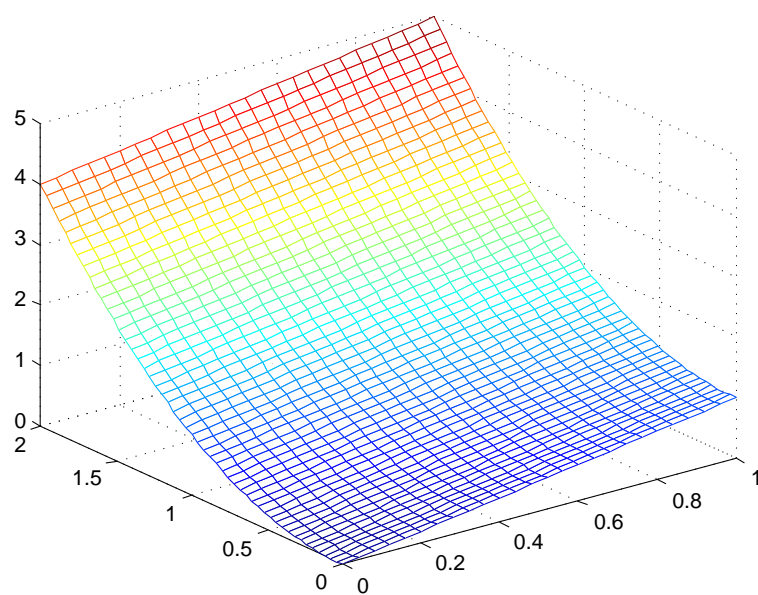


Figure 1: The surface  $z = x + y^2$ ,  $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$ .

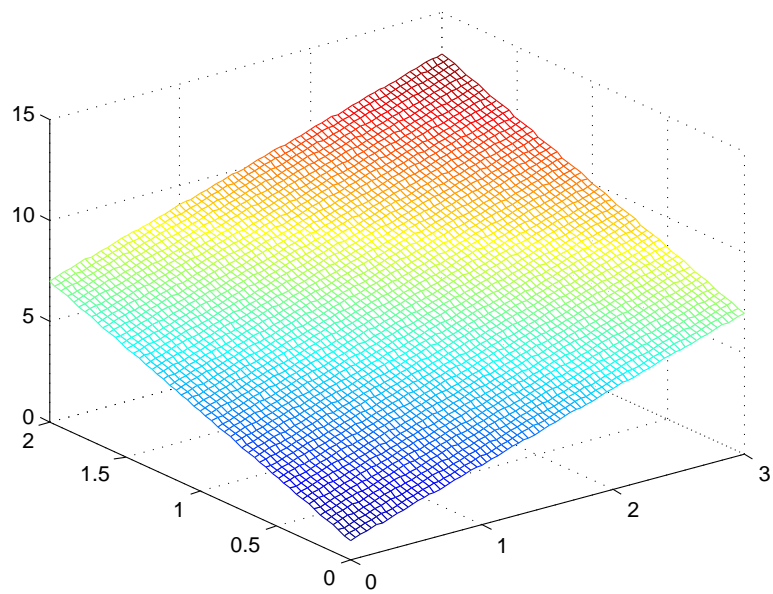


Figure 2: The portion of the plane  $z = 1 + 2x + 3y$  that lies above  $0 \leq x \leq 3$  and  $0 \leq y \leq 2$ .