Math 2401 (K1-K3)
04/01/2015

Worksheet 15 - Conservative Fields

1. Find a potential function for the field:

$$\mathbf{F} = (2xy^3z^4)\mathbf{i} + (3x^2y^2z^4)\mathbf{j} + (4x^2y^3z^3)\mathbf{k}.$$

2. Find a potential function for the field:

$$\mathbf{F} = \left(2x\cos(y) - 2z^3\right)\mathbf{i} + \left(3 + 2ye^z - x^2\sin(y)\right)\mathbf{j} + \left(y^2e^z - 6xz^2\right)\mathbf{k}.$$

3. Consider the vector field:

$$\mathbf{F} = (2x^3y^4 + x)\mathbf{i} + (2x^4y^3 + y)\mathbf{j}.$$

a). Use the Component Test to determine if the field is conservative.

b). If so, find a potential function for $\mathbf{F}$.

c). Use the Fundamental Theorem to find $\int_C \mathbf{F}\cdot d\mathbf{r}$, where

$$\mathbf{r}(t) = (t\cos(\pi t) - 1)\,\mathbf{i} + \sin\left(\frac{\pi t}{2}\right)\mathbf{j},\;\; 0 \le t \le 1.$$

d). Try to set up $\int_C \mathbf{F}\cdot d\mathbf{r}$ the "old way," to convince yourself how much more complicated that would be.

4. Consider the vector field:

$$\mathbf{F} = (e^x\cos y + yz)\,\mathbf{i} + (xz - e^x\sin y)\,\mathbf{j} + (xy + z)\,\mathbf{k}.$$

a). Check that $\mathbf{F}$ is conservative by showing that $\text{curl}\mathbf{F} = \mathbf{0}$.

b). Find a potential function for $\mathbf{F}$.

c). Find $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is any "nice" path (i.e. piecewise smooth, simple, positively oriented) from $(0,0,0)$ to $(0,\pi,1)$.

d). Find $\oint \mathbf{F}\cdot d\mathbf{r}$ where C is any "nice" loop in space.

5. Find a potential function for:

$$\mathbf{F} = \left\langle 2\cos y,\; \frac{1}{y} - 2x\sin y,\; \frac{1}{z}\right\rangle.$$

6. Find a potential function for:

$$\begin{aligned}\mathbf{F} \;=\; & \left(-2xy^2\sin(x^2y^2)\sin(z) + y\cos(xy)e^{\sin(xy)}z\right)\mathbf{i}\\ & + \left(-2x^2y\sin(x^2y^2)\sin(z) + x\cos(xy)e^{\sin(xy)}z\right)\mathbf{j}\\ & + \left(\cos(x^2y^2)\cos(z) + e^{\sin(xy)}\right)\mathbf{k}.\end{aligned}$$

Hint: Remember that you do not *have* to start with the first component...