Math 2401 (K1-K3)
03/30/2015

Worksheet 14 - Flow, Circulation and Flux

1. Find the work done by the force field $\mathbf{F}(x,y) = x^2\mathbf{i} - xy\mathbf{j}$ to move a particle along the quarter circle $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$, $0 \le t \le \frac{\pi}{2}$.

2. Find the flow of the field $\mathbf{F}(x,y,z) = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$ along the curve C: $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$, $0 \le t \le 1$.

3. Find the flow of the field $\mathbf{F}(x,y) = x^2y^3\mathbf{i} - y\sqrt{x}\mathbf{j}$ along the curve C: $\mathbf{r}(t) = t^2\mathbf{i} - t^3\mathbf{j}$, $0 \le t \le 1$.

4. Find the flow of the field $\mathbf{F}(x,y,z) = \sin(x)\mathbf{i} + \cos(y)\mathbf{j} + xz\mathbf{k}$ along the curve C: $\mathbf{r}(t) = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}$, $0 \le t \le 1$.

5. Consider the vector field $\mathbf{F}(x,y) = (x-y)\mathbf{i} + (xy)\mathbf{j}$ (see Figure 1).

a). Find the flow of $\mathbf{F}$ along C_1, where C_1 is the arc of the circle $x^2 + y^2 = 4$ traversed counterclockwise from $(2,0)$ to $(0,-2)$.

b). Find the flow of $\mathbf{F}$ along C_2, where C_2 is the arc of the circle $x^2 + y^2 = 4$ traversed counterclockwise from $(0,-2)$ to $(2,0)$.

c). Find the *circulation* of $\mathbf{F}$ around the circle C: $x^2 + y^2 = 4$.

d). Find the *flux* of $\mathbf{F}$ around the circle C: $x^2 + y^2 = 4$.

6. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$, where

$$\mathbf{F}(x,y) = \left\langle e^{x-1}, xy \right\rangle$$

and $\mathbf{r}(t) = \left\langle t^2, t^3 \right\rangle$, $0 \le t \le 1$.

7. Find the work done by the force field $\mathbf{F}(x,y) = \langle x, y+2 \rangle$ to move a particle along an arch of the cycloid:

$$\mathrm{C}: \ \mathbf{r}(t) = \langle t - \sin(t), 1 - \cos(t) \rangle \, ; \ 0 \le t \le 2\pi.$$

(See Figure 2).

8. Figure 3 shows a plot of the so-called "vortex" vector field:

$$\mathbf{F}(x,y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle.$$

Find the circulation and the flux of this field around the circle C, centered at the origin, of radius 2, traversed counterclockwise.

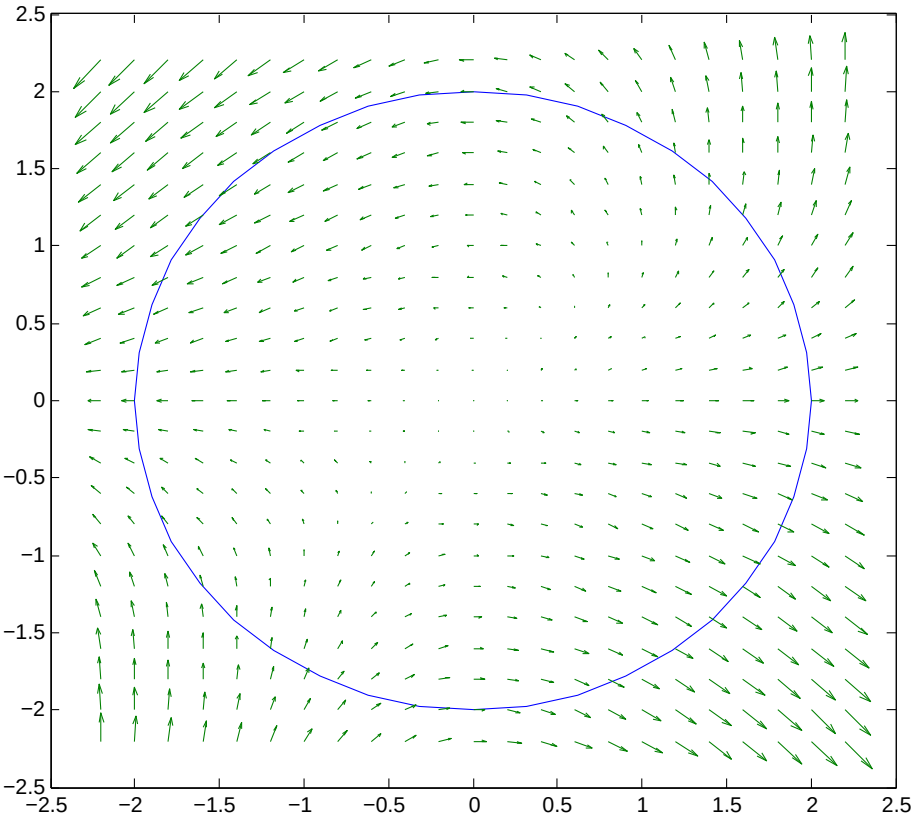

Figure 1: The vector field $\mathbf{F}$ and the curve C in Problem 5.

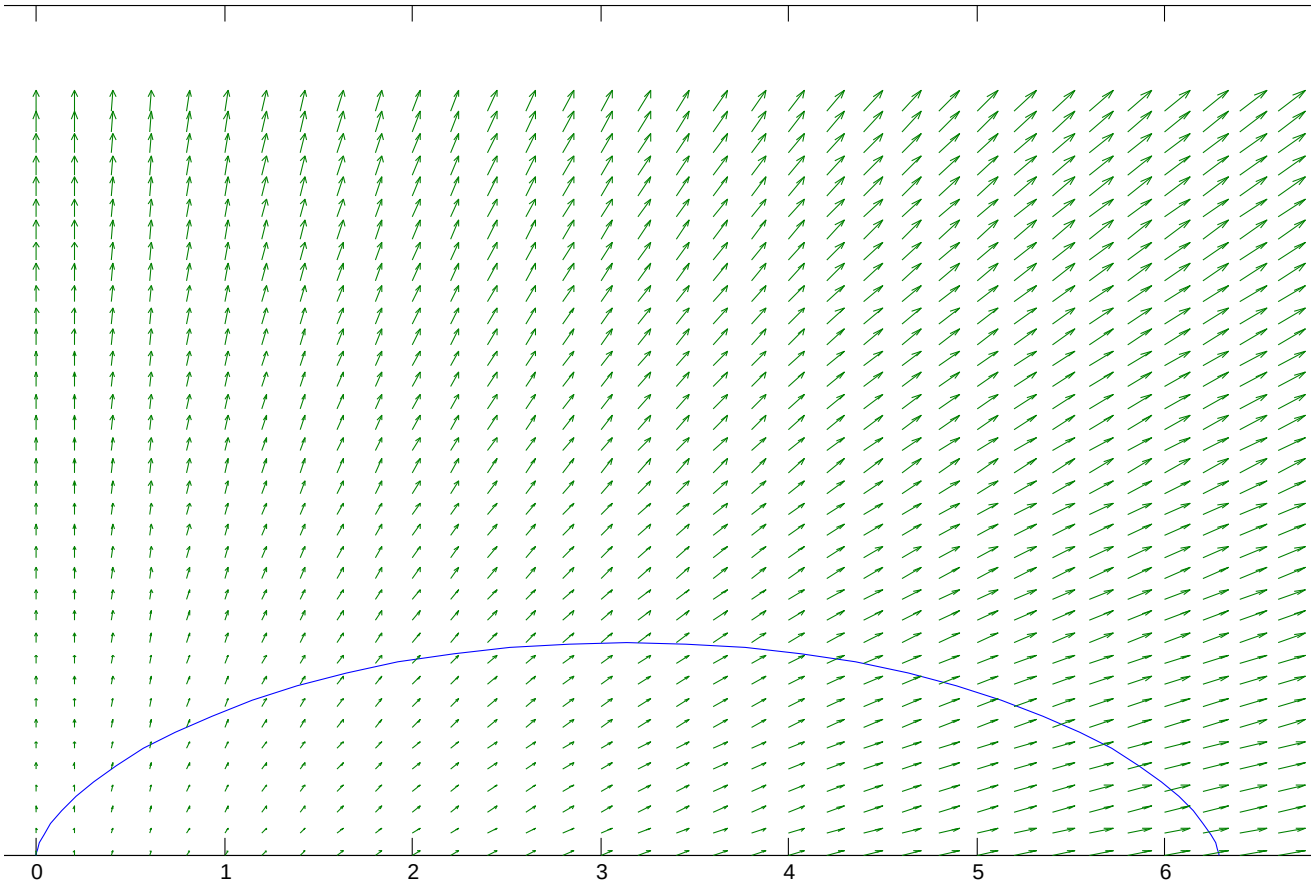

Figure 2: The vector field $\mathbf{F}$ and the curve C in Problem 7.

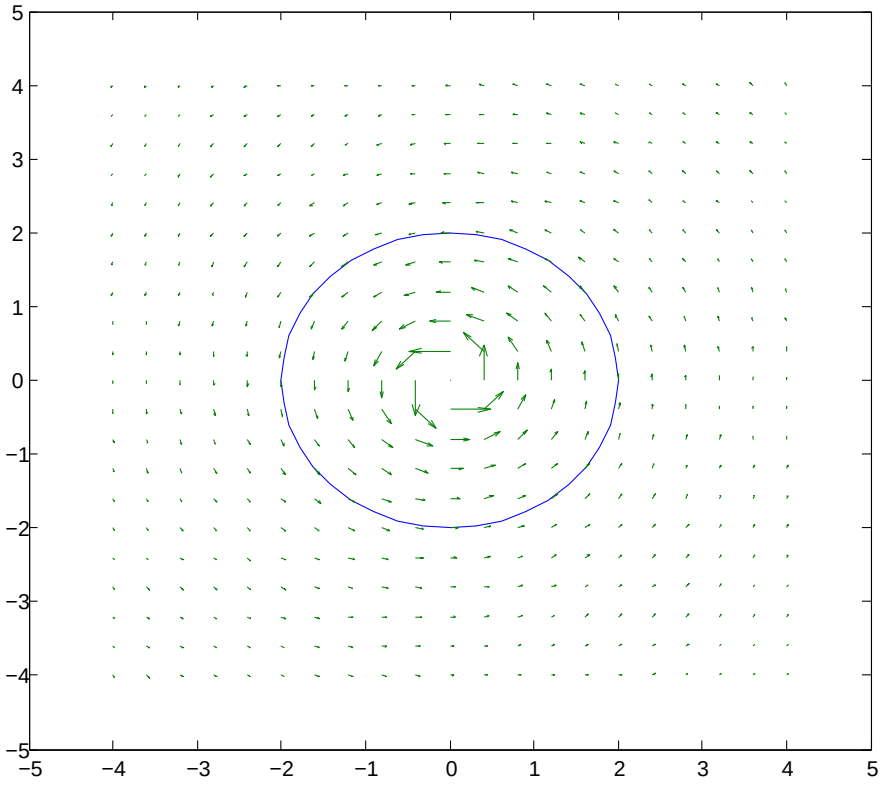

Figure 3: The "vortex" vector field $\mathbf{F}$ and the curve C in Problem 8.