Worksheet 12 - More Exam 3 Review Problems

1. Consider:

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\,dy\,dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\,dy\,dx.$$

Note that the integrand is the same in all three integrals, namely xy, only the regions of integration differ. Sketch these regions and "put them together," that is, express this as one integral:

$$\iint_R xy\,dy\,dx,$$

over the appropriate region R. You should note then that the resulting region is easily described using polar coordinates, so change to polar coordinates and compute the value of the expression above.

2. Let R be the region in the first quadrant that is interior to the rectangle with vertices $(0,0)$, $(3\sqrt{3},0)$, $(3\sqrt{3},3)$, and $(0,3)$, and exterior to the unit circle centered at the origin. Describe the region R in polar coordinates.

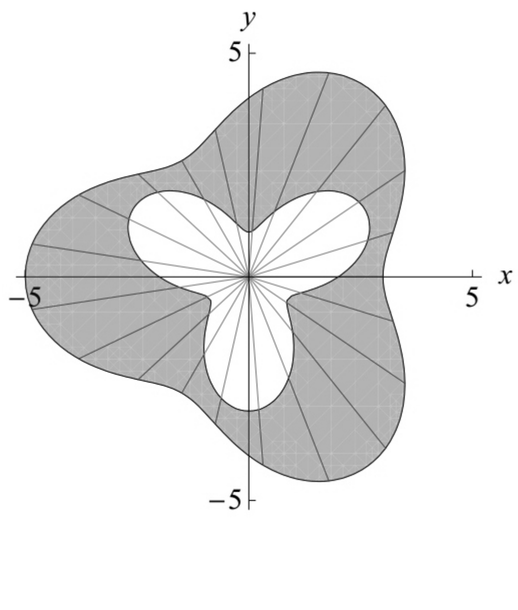
3. Compute the integrals:

$$\text{a).}\quad \int_{\pi/2}^{\pi}\int_{0}^{x^2} \frac{1}{x}\cos\frac{y}{x}\,dy\,dx$$

$$\text{b).}\quad \int_{1}^{4}\int_{0}^{\sqrt{y}} e^{x/\sqrt{y}}\,dx\,dy$$

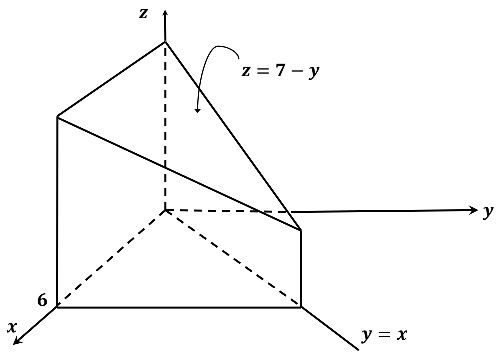
$$\text{c).}\quad \iint_R \frac{\sin x}{x}\,dA, \text{ where: } R = \{(x,y) : 0 \le y \le x; 0 \le x \le \pi\}.$$

4. Consider the region R, pictured below, which is the region between the polar curves $r = 2 + \sin(3\theta)$ - the inner curve - and $r = 4 - \cos(3\theta)$ - the outer curve. Find the area of this region.

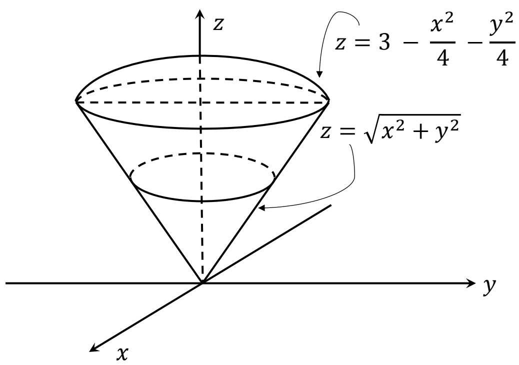


5. Set up the triple integral in cylindrical coordinates to compute the volume of the prism (pictured below) with

- Base: triangle in the x, y plane bounded by the x-axis, $y = x$, and $x = 6$.
- Top: The plane $z = 7 - y$.



6. Find the volume of the "ice-cream cone" pictured below:



7. Find the area of the region that lies inside of $x^2 + (y - 1)^2 = 1$ and outside of $x^2 + y^2 = 1$.