

Worksheet 10
Substitution in Double Integrals

1. Use substitution to find:

$$\iint_R e^{4x-y} dA,$$

where R is the parallelogram with vertices $(0, 0)$, $(3, 3)$, $(7, 4)$, and $(4, 1)$.

Hint: Try the substitution $u = x - y$; $v = x - 4y$.

Hint: There are often more than one “good” substitutions in these cases; for instance, try also the substitution $u = 4x - y$ and $v = x - y$.

2. Use substitution to find:

$$\iint_R (x^2 + y^2) dA,$$

where R is the domain bounded by $1 \leq xy \leq 4$ and $1 \leq \frac{y}{x} \leq 4$.

Hint: Try $u = xy$ and $v = \frac{y}{x}$.

3. Use substitution to find:

$$\iint_R (x + y)^2 \sin^2(x - y) dA,$$

where R is the square with vertices $(0, 1)$, $(1, 2)$, $(2, 1)$, and $(1, 0)$.

Hint: Try a substitution dictated by making the integral easier to compute.

4. Compute the integral:

$$\int_1^2 \int_{x+2}^{x+3} \frac{dy dx}{\sqrt{xy - x^2}}$$

by using the substitution $u = x$; $v = y - x$. Try to also compute the integral as it was given.

5. (a). Compute the integral:

$$\int_0^4 \sqrt{x} \cos(\sqrt{x}) dx$$

by using the substitution $u = \sqrt{x}$.

- (b). Verify that

$$\int_0^4 \sqrt{x} \cos(\sqrt{x}) dx = \int_0^4 \int_0^1 \sqrt{x} \cos(\sqrt{x}) dy dx$$

and that you get the same result as in part (a). by working with this double integral and using the substitution $u = \sqrt{x}$, $v = y$.