NAME: Solutions

Math 2401 (K1-K3)

Quiz 9
The Last Quiz!!!

(5pts.)

1. (a). Find a potential function for the conservative field:

$$\mathbf{F}(x, y, z) = 18x^2\mathbf{i} + \frac{4z^2}{y}\mathbf{j} + 8z\ln(y)\mathbf{k}.$$

(b). Use part (a). to compute:

$$\int_{(6,1,1)}^{(6,5,3)} 18x^2\,dx + \frac{4z^2}{y}\,dy + 8z\ln(y)\,dz.$$

(5pts.)

2. Recall Green's formulas:

$$\boxed{\oint_C \vec{F}\cdot\vec{n}\,ds = \oint_C M\,dy - N\,dx = \int\int_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right)\,dA},$$

$$\boxed{\oint_C \vec{F}\cdot\vec{T}\,ds = \oint_C M\,dx + N\,dy = \int\int_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\,dA}.$$

Use this to find:

$$\oint_C \left(2y + \sqrt{1+x^5}\right)\,dx + \left(5x - e^{y^2}\right)\,dy,$$

where C is a positively oriented rectangle in the plane, with sides of length 2 and 5.

1 (a). $\frac{\partial f}{\partial x} = 18x^2 \Rightarrow f(x,y,z) = 6x^3 + g(y,z)$

(3 pts.)
1pt./component

$\Rightarrow \left.\begin{aligned}\frac{\partial f}{\partial y} &= \frac{\partial g}{\partial y}\\ &= \frac{4z^2}{y}\end{aligned}\right\} \Rightarrow \frac{\partial g}{\partial y} = \frac{4z^2}{y} \Rightarrow g(y,z) = 4z^2\ln(y) + h(z)$

$\Rightarrow f = 6x^3 + 4z^2\ln(y) + h(z)$

$\Rightarrow \left.\begin{aligned}\frac{\partial f}{\partial z} &= 8z\ln(y) + h'(z)\\ &= 8z\ln(y)\end{aligned}\right\} \Rightarrow h'(z) = 0 \Rightarrow h(z) = C$

$$\boxed{f(x,y,z) = 6x^3 + 4z^2\ln(y) + C}$$

(2pts.)

(b). $\int_{(6,1,1)}^{(6,5,3)} \vec{F}\cdot d\vec{r} = f(6,5,3) - f(6,1,1) = \left(\cancel{6^4} + 4\cdot 9\ln(5)\right) - \left(\cancel{6^4} + \underbrace{4\ln(1)}_{0}\right)$

1pt. for correct setup

$= \boxed{36\ln(5)}$ (1pt. for computation)

(2) $\oint_C \left(2y+\sqrt{1+x^5}\right)dx + \left(5x-e^{y^2}\right)dy$

$= \oint_C M\,dx + N\,dy$

$M = 2y + \sqrt{1+x^5}$

$N = 5x - e^{y^2}$

$= \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$

$= \iint_R (5-2)\,dA$

$= \iint_R 3\,dA$

$= 3 \cdot \underbrace{\text{Area}(R)}_{10} = \boxed{30}$