NAME: Solutions

Math 2401 (K1-K3)
2/11/2015

Quiz 4

(4 pts.) **1.** Let $f(x,y) = \cos^2(3x - y^2)$. Find $2yf_x + 3f_y$.

(3 pts.) **2.** Find the directions in which the directional derivative of

$$f(x,y) = \sin(y) + \cos(xy) + e^x$$

at the point $(0,0)$ has the value 1.

(3 pts.) **3.** If

$$g(s,t) = f\left(e^{s-t}, e^{t-s}\right),$$

where f is a differentiable function, find:

$$\frac{\partial g}{\partial s} + \frac{\partial g}{\partial t}.$$

Hint: Don't panic because you don't know exactly what the partial derivatives of f look like - it works out.

① $f_x = 2\cos(3x-y^2)\left(-\sin(3x-y^2)\cdot 3\right) = -6\cos(3x-y^2)\sin(3x-y^2)$ (1½ pts.)

$f_y = 2\cos(3x-y^2)\left(-\sin(3x-y^2)\cdot(-2y)\right) = 4y\cos(3x-y^2)\sin(3x-y^2)$ (1½ pts.)

$2yf_x = -12y\cos(3x-y^2)\sin(3x-y^2)$

$3f_y = 12y\cos(3x-y^2)\sin(3x-y^2)$

$2yf_x + 3f_y = \boxed{0}$ (1 pt.)

② $\nabla f = \langle -y\sin(xy) + e^x, \cos(y) - x\sin(xy)\rangle$ (1 pt.)

$\nabla f(0,0) = \langle 1, 1\rangle$ (½ pt.)

$D_{\vec{u}} f\big|_{(0,0)} = 1$; $\vec{u} = \langle a,b\rangle \Rightarrow \nabla f(0,0)\cdot\vec{u} = 1 \Rightarrow \langle 1,1\rangle\cdot\langle a,b\rangle = 1$

$\Rightarrow \begin{cases} a+b=1 \\ a^2+b^2=1 \end{cases}$ (½ pt.) $\Rightarrow \begin{cases} b = 1-a \\ a^2+(1-a)^2 = 1 \end{cases} \Rightarrow a^2 + \cancel{1} - 2a + a^2 = \cancel{1}$

$2a^2 - 2a = 0 \Rightarrow 2a(a-1) = 0$

$\Rightarrow a = 0$ or $a = 1$
$b = 1$ $\quad$ $b = 0$

$\boxed{\vec{u} = \langle 0,1\rangle}$ or $\boxed{\vec{u} = \langle 1,0\rangle}$

(½ pt.) (½ pt.)

③ $g(s,t) = f(e^{s-t}, e^{t-s})$

$$\left.\begin{aligned} x(s,t) &= e^{s-t} \\ y(s,t) &= e^{t-s} \end{aligned}\right\} \Rightarrow g(s,t) = f(x,y) \quad (\tfrac{1}{2}\text{pt.})$$

$$\frac{\partial g}{\partial s} = \frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$

$$= \frac{\partial f}{\partial x} e^{s-t} + \frac{\partial f}{\partial y} e^{t-s}(-1) \quad (1\text{pt.})$$

$$\frac{\partial g}{\partial t} = \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$$

$$= \frac{\partial f}{\partial x} e^{s-t}(-1) + \frac{\partial f}{\partial y} e^{t-s} \quad (1\text{pt.})$$

$$\Rightarrow \frac{\partial g}{\partial s} = \frac{\partial f}{\partial x} e^{s-t} - \frac{\partial f}{\partial y} e^{t-s}$$

$$\frac{\partial g}{\partial t} = -\frac{\partial f}{\partial x} e^{s-t} + \frac{\partial f}{\partial y} e^{t-s}$$

$$\oplus \quad \frac{\partial g}{\partial s} + \frac{\partial g}{\partial t} = \boxed{0}. \quad (\tfrac{1}{2}\text{pt.})$$