Math 2401

Chapter $\boxed{12}$ Review

- **Distance formula**: distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$: $\boxed{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$
- **Equation of sphere**: with radius R and center (x_0, y_0, z_0): $\boxed{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2}$
- **Magnitude (length) of a vector**: length of $\vec{v}=\langle v_1, v_2, v_3\rangle$: $\boxed{|\vec{v}|=\sqrt{v_1^2+v_2^2+v_3^2}}$
- **Direction of a vector**: direction of $\vec{v}$: $\boxed{\dfrac{\vec{v}}{|\vec{v}|}}$ when $\vec{v}\neq\vec{0}$.
- **Dot Product**: dot product of $\vec{u}=\langle u_1, u_2, u_3\rangle$ and $\vec{v}=\langle v_1, v_2, v_3\rangle$: $\boxed{\vec{u}\cdot\vec{v}=u_1v_1+u_2v_2+u_3v_3}$
- **Angle b/w vectors**: angle θ $(0\leq\theta\leq\pi)$ between $\vec{u}$ and $\vec{v}$: $\boxed{\theta=\cos^{-1}\left(\dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)}$
- **Dot Product Properties**:

  $\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$

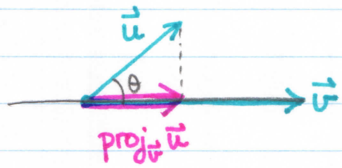
  $(c\vec{u})\cdot\vec{v}=c(\vec{u}\cdot\vec{v})$

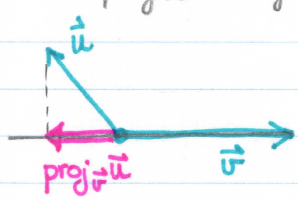
  $\vec{u}\cdot(\vec{v}+\vec{w})=\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}$

  $\boxed{\vec{u}\cdot\vec{u}=|\vec{u}|^2}$

  $\boxed{\boxed{\vec{u}\perp\vec{v}} \iff \boxed{\vec{u}\cdot\vec{v}=0}}$

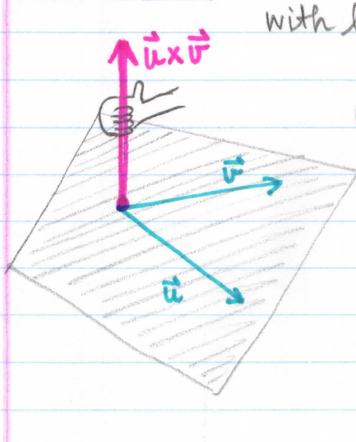
- **Vector Projection**: The vector projection of $\vec{u}$ onto $\vec{v}$ $(\vec{v}\neq 0)$:





$$\boxed{\begin{aligned}\text{proj}_{\vec{v}}\vec{u} &= \underbrace{(|\vec{u}|\cos\theta)}_{\text{"scalar component of } \vec{u} \text{ in the direction of } \vec{v}\text{"}}\frac{\vec{v}}{|\vec{v}|} \\ &= \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}\end{aligned}}$$

- **Cross Product**: $(\vec{u}\times\vec{v})$ is a vector perpendicular to the plane determined by $\vec{u}$ & $\vec{v}$, with length: $\boxed{|\vec{u}\times\vec{v}|=\underbrace{|\vec{u}||\vec{v}||\sin\theta|}_{\text{area of parallelogram determined by } \vec{u} \,\&\, \vec{v}}}$



(Right Hand Rule)

Determinant Form of Cross Product:

$$\boxed{\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix}}$$

- **Properties of Cross Product**

$$(a\vec{u})\times(b\vec{v}) = (ab)(\vec{u}\times\vec{v})$$

$$\boxed{\vec{v}\times\vec{u} = -(\vec{u}\times\vec{v})}$$

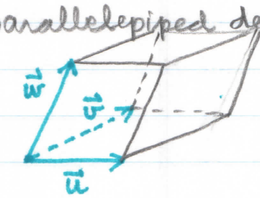
$$\vec{u}\times(\vec{v}+\vec{w}) = \vec{u}\times\vec{v} + \vec{u}\times\vec{w}$$

$$(\vec{u}+\vec{v})\times\vec{w} = \vec{u}\times\vec{w} + \vec{v}\times\vec{w}$$

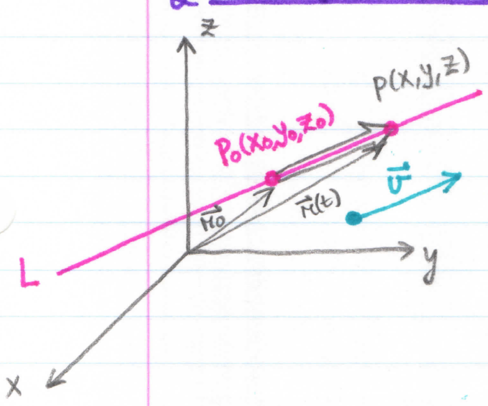
- **Triple Scalar (Box) Product** :

$$\boxed{(\vec{u}\times\vec{v})\cdot\vec{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}}$$

→ Volume of parallelepiped determined by $\vec{u}, \vec{v}, \vec{w}$.



- **Equations for a Line in Space**



L : The line going through the point $P_0(x_0, y_0, z_0)$ and parallel to the vector $\vec{v} = \langle v_1, v_2, v_3\rangle$:

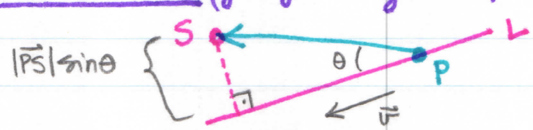
Vector Eqn.: $\boxed{\vec{r}(t) = \vec{r}_0 + t\vec{v}}$

Parametric Eqns.:

$$\boxed{\begin{aligned} x &= x_0 + tv_1 \\ y &= y_0 + tv_2 \\ z &= z_0 + tv_3 \end{aligned}}$$

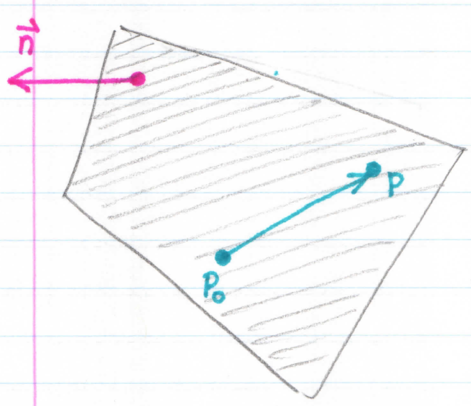
- **Distance from a point S to a line (going through P & parallel to $\vec{v}$)** :

$$\boxed{d = \frac{|\vec{PS}\times\vec{v}|}{|\vec{v}|}}$$



- **Equations for a Plane in Space**

The plane going through the point $P_0(x_0, y_0, z_0)$ with a normal vector $\vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$:



Vector Eqn.: $\boxed{\vec{n}\cdot\vec{P_0P} = \vec{0}}$

Component Eqn.: $\boxed{A(x-x_0) + B(y-y_0) + C(z-z_0) = 0}$

Simplified Component Eqn.: $\boxed{Ax + By + Cz = D, \text{ w/ } D = Ax_0 + By_0 + Cz_0}$