

Name: _____

September 23rd, 2015.
Math 2552; Sections F1 – F4; L1 – L4.
Georgia Institute of Technology
Sample Exam 1

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged: _____

Problem	Possible Score	Earned Score
1	16	
2	17	
3	17	
4	17	
5	17	
6	16	
Total	100	

Remember that you must **SHOW YOUR WORK** to receive credit!

Good luck!

1. Consider the autonomous equation:

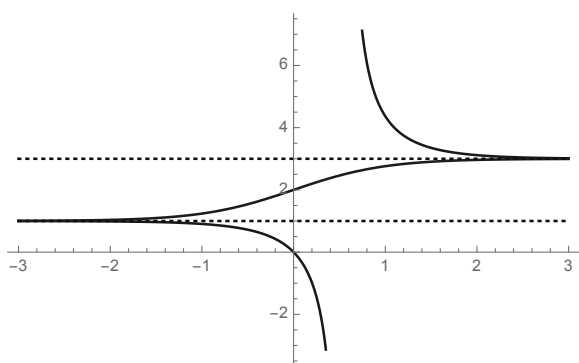
$$\frac{dy}{dx} = (y - 1)(y - 3).$$

(a). Find the equilibrium solutions:

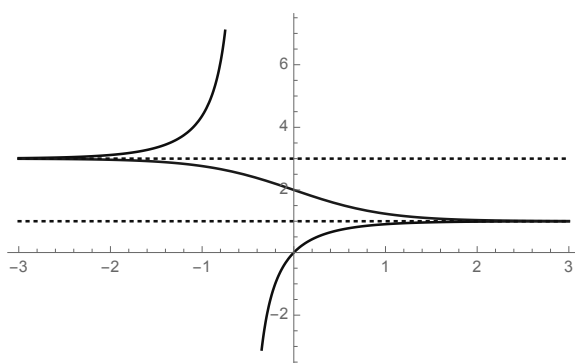
(b). Draw the phase portrait.

(c). Determine which of the graphs below could be possible solutions to this equation (circle the correct one).

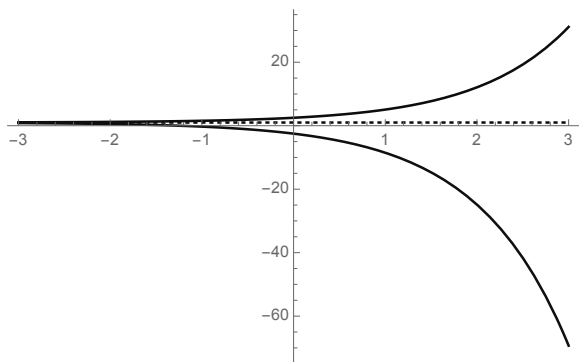
A.



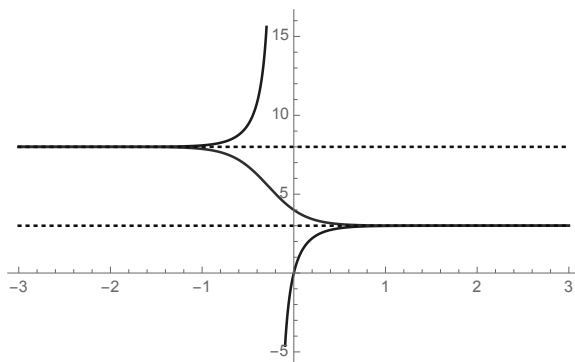
B.



C.



D.



2. Find an *explicit* solution to the initial value problem:

$$\frac{dy}{dx} = e^x + y; \quad y(0) = 0,$$

and state the largest interval where your solution is valid.

3. Consider the differential equation:

$$(y^2 - x^2) dx - 2xy dy = 0.$$

(a). Determine whether or not the equation is exact.

(b). Solve the equation (give an *implicit* solution).

4. Suppose that $y(x)$ is the solution to the initial value problem:

$$\frac{1}{x}e^{y^2} \frac{dy}{dx} = \frac{1}{y(1+x^2)}; \quad y(0) = 0.$$

Find

$$e^{y^2(1)}.$$

5. Solve the differential equation:

$$y' = \frac{2 + ye^{xy}}{2y - xe^{xy}}.$$

Give an *implicit* solution.

6. Given that the general solution to the differential equation:

$$\frac{du}{dt} - \frac{2}{t}u = 8t^2 \cos t$$

is:

$$u(t) = 8t^2 \sin t + ct^2,$$

find the solution to the differential equation:

$$y \frac{dy}{dx} - \frac{1}{x}y^2 = 4x^2 \cos x.$$