Math 2552 - Differential Equations **Homework 6**
Sections F1 – F4; L1 – L4
Georgia Institute of Technology, Fall 2015

Higher Order Linear ODEs

1. Given that:

$$y = c_1 e^x \cos x + c_2 e^x \sin x$$

is a two-parameter family of solutions to the equation

$$y'' - 2y' + 2y = 0,$$

determine for each BVP below if there exists a solution, no solutions, or several solutions.

a). $y(0) = 1;\ y'(0) = 0.$

b). $y(0) = 1;\ y(\pi) = -1.$

c). $y(0) = 0;\ y(\pi) = 0.$

d). $y(0) = 1;\ y(\pi/2) = 1.$

2. For each of the sets of functions below, show that they are linearly dependent.

a). $f_1(x) = 5;\ f_2(x) = \cos^2 x;\ f_3(x) = \sin^2 x.$

b). $f_1(x) = \cos(2x);\ f_2(x) = 1;\ f_3(x) = \cos^2 x.$

c). $f_1(x) = x;\ f_2(x) = x^2;\ f_3(x) = 4x - 3x^2.$

d). $f_1(x) = x;\ f_2(x) = \sin x;\ f_3(x) = 3x;\ f_4(x) = e^x.$

3. Show that each set of functions below is linearly independent on the given interval, by computing their Wronskian.

a). $\sqrt{x},\ x^2;\ (0, \infty).$

b). $1 + x,\ x^3;\ (-\infty, \infty).$

c). $e^x,\ e^{-x},\ e^{4x};\ (-\infty, \infty).$

d). $x,\ x \ln x,\ x^2 \ln x;\ (0, \infty).$

e). $e^x \cos(2x),\ e^x \sin(2x),\ e^x;\ (-\infty, \infty).$

4. For each of the equations below, verify that the given set of functions is a *fundamental set* of solutions on the given interval, and write the general solution.

a). $y'' - y' - 12y = 0;\ \{e^{-3x},\ e^{4x}\};\ (-\infty, \infty).$

b). $y^{(4)} + y'' = 0;\ \{1,\ x,\ \cos x,\ \sin x\};\ (-\infty, \infty).$

5. For each of the equations below, verify that the given function is a particular solution. Use the results in problem 4 to write the general solution.

a). $y'' - y' - 12y = \sin x - 13 \cos x;\ y_p = \cos x.$

b). $y^{(4)} + y'' = 20e^{2x};\ y_p = e^{2x}.$