Math 2552 - Differential Equations **Homework 2**

Sections F1 – F4; L1 – L4

Georgia Institute of Technology, Fall 2015

Exact Differential Equations

For each ODE below, determine whether or not it is exact. If it is exact, solve it. (No need to give an interval of validity.)

1. $(5x + 4y)\,dx + (4x - 8y^3)\,dy = 0.$

2. $\big(\sin y - y\sin x\big)\,dx + \big(\cos x + x\cos y - y\big)\,dy = 0.$

3. $\big[\cos(xy) - xy\sin(xy)\big]\,dx - x^2\sin(xy)\,dy = 0.$

4. $ye^{xy}\,dx + (2y - xe^{xy})\,dy = 0.$

5. $\big(1 + \ln(xy)\big)\,dx + \dfrac{x}{y}\,dy = 0.$

6. $(2y^2x - 3)\,dx + (2yx^2 + 4)\,dy = 0.$

7. $\big(2y - \dfrac{1}{x} + \cos(3x)\big)\dfrac{dy}{dx} + \dfrac{y}{x^2} - 4x^3 + 3y\sin(3x) = 0.$

8. $(x^3 + y^3)\,dx + 3xy^2\,dy = 0.$

9. $\big(y^3 - y^2\sin x - x\big)\,dx + \big(3xy^2 + 2y\cos x\big)\,dy = 0.$

10. $\big(y\ln y - e^{-xy}\big)\,dx + \big(\dfrac{1}{y} + x\ln y\big)\,dy = 0.$

11. $\dfrac{2x}{y}\,dx - \dfrac{x^2}{y^2}\,dy = 0.$

12. $\Big(\dfrac{1}{x} - \dfrac{y}{x^2 + y^2}\Big)\,dx + \dfrac{x}{x^2 + y^2}\,dy = 0.$

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Solve each of the initial value problems below:

13. $(x + y)^2\,dx + (2xy + x^2 - 1)\,dy = 0;\;\; y(1) = 1.$

14. $(e^x + y)\,dx + (2 + x + ye^y)\,dy = 0;\;\; y(0) = 1.$

15. $(y^2\cos x - 3x^2y - 2x)\,dx + (2y\sin x - x^3 + \ln y)\,dy = 0;\;\; y(0) = e.$

16. $\Big(\dfrac{1}{1 + y^2} + \cos x - 2xy\Big)\dfrac{dy}{dx} = y(y + \sin x);\;\; y(0) = 1.$

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For each of the ODEs below, find the value of k such that the equation is exact, and then solve the equation.

17. $(y^3 + kxy^4 - 2x)\,dx + (3xy^2 + 20x^2y^3)\,dy = 0.$

18. $(2x - y\sin(xy) + ky^4)\,dx - (20xy^3 + x\sin(xy))\,dy = 0.$

19. $(2xy^2 + ye^x)\,dx + (2x^2y + ke^x - 1)\,dy = 0.$

20. $(6xy^3 + \cos y)\,dx + (kx^2y^2 - x\sin y)\,dy = 0.$

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Find a function $M(x, y)$ such that the ODE is exact:

21. $M(x, y)\,dx + \Big(xe^{xy} + 2xy + \dfrac{1}{x}\Big)\,dy = 0.$

Find a function $N(x, y)$ such that the ODE is exact:

22. $\left(\dfrac{\sqrt{y}}{\sqrt{x}} + \dfrac{x}{x^2 + y}\right)dx + N(x, y)\,dy = 0.$

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For the ODEs below, verify that the given $\mu(x, y)$ is an integrating factor, and use it to solve the equation:

23. $y(x + y + 1)\,dx + (x + 2y)\,dy = 0;\;\; \mu(x, y) = e^x.$

24. $(-xy\sin x + 2y\cos x)\,dx + 2x\cos x\,dy = 0;\;\; \mu(x, y) = xy.$