Inverse Laplace Transform

① $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{1}{2}t^2$

② $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{48}{s^5}\right\} = t - 48\,\frac{1}{4!}t^4 = t - 2t^4$

③ $\mathcal{L}^{-1}\left\{\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2}\right\} = t - 1 + e^{2t}$

④ $\mathcal{L}^{-1}\left\{\frac{(s+1)^3}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{s^3+3s^2+3s+1}{s^4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + 3\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^4}\right\}$

$= 1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3.$

⑤ $\mathcal{L}^{-1}\left\{\frac{5}{s^2+49}\right\} = \frac{5}{7}\sin(7t).$

⑥ $\mathcal{L}^{-1}\left\{\frac{1}{s^2-16}\right\} = \frac{1}{4}\sinh(4t).$

⑦ $\mathcal{L}^{-1}\left\{\frac{1}{4s+1}\right\} = \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+1/4}\right\} = \frac{1}{4}e^{-1/4\,t}$

⑧ $\mathcal{L}^{-1}\left\{\frac{4s}{4s^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1/4}\right\} = \cos\left(\frac{1}{2}t\right).$

⑨ $\mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+9}\right\} = 2\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 6\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\} = 2\cos(3t) - 2\sin(3t).$

⑩ $\mathcal{L}^{-1}\left\{\frac{1}{s^2+3s}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s+3)}\right\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} = \frac{1}{3} - \frac{1}{3}e^{-3t}.$

$\frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$; $1 = A(s+3) + Bs$

$s=0 \Rightarrow \boxed{A = 1/3}$

$s=-3 \Rightarrow \boxed{B = -1/3}$

⑪ $\mathcal{L}^{-1}\left\{\frac{s}{s^2+2s-3}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+3)(s-1)}\right\} = \frac{3}{4}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} + \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\} = \frac{3}{4}e^{-3t} + \frac{1}{4}e^{t}$

$\frac{s}{(s+3)(s-1)} = \frac{A}{s+3} + \frac{B}{s-1}$

"Cover up" $(s-1)$: $\left.\frac{s}{s+3}\right|_{s=1} = B \Rightarrow \boxed{B = 1/4}$

"Cover up" $(s+3)$: $\left.\frac{s}{s-1}\right|_{s=-3} = A \Rightarrow \boxed{A = 3/4}$

⑫ $\mathcal{L}^{-1}\left\{\frac{3s}{s+4}\right\}$ <u>DNE</u>

$\lim_{s\to\infty} \frac{3s}{s+4} = 3 \neq 0.$

(13) $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)(s-3)(s-6)}\right\} = \frac{1}{2}e^{2t} - e^{3t} + \frac{1}{2}e^{6t}$

$$\frac{s}{(s-2)(s-3)(s-6)} = \frac{A}{s-2} + \frac{B}{s-3} + \frac{C}{s-6}$$

$$\left.\frac{s}{(s-3)(s-6)}\right|_{s=2} = A \Rightarrow A = 1/2 \; ; \quad \left.\frac{s}{(s-2)(s-6)}\right|_{s=3} = B \Rightarrow B = -1$$

$$\left.\frac{s}{(s-2)(s-3)}\right|_{s=6} = C \Rightarrow C = 1/2$$

(14) $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\} = \frac{1}{4}t - \frac{1}{8}\sin(2t).$

$$\frac{1}{s^2(s^2+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+4} \qquad 1 = As(s^2+4) + B(s^2+4) + (Cs+D)s^2$$

"Cover up" s^2: $\left.\frac{1}{s^2+4}\right|_{s=0} = B \Rightarrow B = 1/4$

$$1 = As(s^2+4) + \frac{1}{4}(s^2+4) + s^2(Cs+D)$$

Coefficients of s^3: $0 = A + C \Rightarrow C = -A$

s^2: $0 = \frac{1}{4} + D \Rightarrow D = -1/4$

s: $0 = 4A \Rightarrow A = 0 \Rightarrow C = 0$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+4)}\right\} =$$
$$= \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} - \frac{1}{4}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$$
$$= \frac{1}{4}t - \frac{1}{4}\cdot\frac{1}{2}\sin(2t).$$

Or:

$$\frac{1}{s^2(s^2+4)} = \frac{A}{s} + \frac{1/4}{s^2} + \frac{Cs+D}{s^2+4}$$

$$\Rightarrow \frac{1}{s^2(s^2+4)} - \frac{1}{4s^2} = \frac{A}{s} + \frac{Cs+D}{s^2+4} \Rightarrow -\frac{1}{4(s^2+4)} = \frac{A}{s} + \frac{Cs+D}{s^2+4}$$

$$\frac{4-(s^2+4)}{4s^2(s^2+4)}$$

$\Rightarrow$ By inspection: $A = 0$; $Cs + D = -\frac{1}{4}$

$\Rightarrow C = 0$; $D = -1/4$.

15 $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)(s^2+4)}\right\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} - \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} = \frac{1}{3}\sin(t) - \frac{1}{6}\sin(2t).$

$$\frac{1}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$$

$$1 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

Not so many options here...
Either: – match coefficients
– give 4 values to s
$\Big\} \Rightarrow$ 4x4 system

Coeff. of
$s^3: \quad 0 = A + C \Rightarrow C = -A$
$s^2: \quad 0 = B + D \Rightarrow D = -B$
$s: \quad 0 = 4A + C \Rightarrow C = -4A$
$s^0: \quad 1 = 4B + D$

$-A = -4A \Rightarrow A = C = 0$

$3B = 1 \Rightarrow B = 1/3 \; ; \; D = -1/3$

Simpler way in this case: $\frac{1}{(s^2+1)(s^2+4)} = \frac{1}{3}\frac{(s^2+4)-(s^2+1)}{(s^2+1)(s^2+4)} = \frac{1}{3}\frac{1}{s^2+1} - \frac{1}{3}\frac{1}{s^2+4}.$

16 $\mathcal{L}^{-1}\left\{\frac{2s+4}{(s-2)(s^2+4s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{2s+4}{(s-2)(s+3)(s+1)}\right\} = \frac{8}{15}e^{2t} - \frac{1}{5}e^{-3t} - \frac{1}{3}e^{-t}.$

$$\frac{2s+4}{(s-2)(s+3)(s+1)} = \frac{A}{s-2} + \frac{B}{s+3} + \frac{C}{s+1}$$

$$\left.\frac{2s+4}{(s+3)(s+1)}\right|_{s=2} = A \Rightarrow A = 8/15 \; ; \; \left.\frac{2s+4}{(s-2)(s+1)}\right|_{s=-3} = B \Rightarrow B = -1/5$$

$$\left.\frac{2s+4}{(s-2)(s+3)}\right|_{s=-1} = C \Rightarrow C = -1/3$$

17 $\mathcal{L}^{-1}\left\{\frac{s}{(s^2+4)(s+2)}\right\} = \frac{1}{4}\cos(2t) + \frac{1}{4}\sin(2t) - \frac{1}{4}e^{-2t}.$

$$\frac{s}{(s^2+4)(s+2)} = \frac{As+B}{s^2+4} + \frac{C}{s+2} \quad \Rightarrow \quad \left.\frac{s}{s^2+4}\right|_{s=-2} = C \Rightarrow C = -1/4$$

$$\Rightarrow \frac{As+B}{s^2+4} = \frac{s}{(s^2+4)(s+2)} + \frac{1}{4(s+2)} = \frac{4s+(s^2+4)}{4(s+2)(s^2+4)} = \frac{(s+2)^2}{4(s+2)(s^2+4)} = \frac{s+2}{4(s^2+4)}$$

$$\Rightarrow \frac{As+B}{s^2+4} = \frac{1/4\,s + 1/2}{s^2+4} \Rightarrow A = 1/4 \; ; \; B = 1/2$$

(18) $\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^3}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^3}\Big|_{s\to s+2}\right\} = e^{-2t}\cdot\frac{1}{2}t^2$

(19) $\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+1}\right\}$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\Big|_{s\to(s+2)}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\Big|_{s\to s+2}\right\}$$

$$= e^{-2t}\cos t - 2e^{-2t}\sin t.$$

(20) $\mathcal{L}^{-1}\left\{\frac{1}{s^2-6s+10}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s-3)^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\Big|_{s\to s-3}\right\} = e^{3t}\sin t.$

(21) $\mathcal{L}^{-1}\left\{\frac{s}{(s+1)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} - \mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2}\right\} = e^{-t} - e^{-t}\cdot t.$

"Easy" way: $\frac{s}{(s+1)^2} = \frac{s+1-1}{(s+1)^2} = \frac{1}{s+1} - \frac{1}{(s+1)^2}$

Classical way: $\frac{s}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$

"Cover up" $(s+1)^2 \Rightarrow s\big|_{s=-1} = B \Rightarrow B=-1$

$\frac{s}{(s+1)^2} = \frac{A}{s+1} - \frac{1}{(s+1)^2} \Rightarrow \frac{s+1}{(s+1)^2} = \frac{A}{s+1} \Rightarrow \frac{1}{s+1} = \frac{A}{s+1} \Rightarrow A=1.$

(22) $\mathcal{L}^{-1}\left\{\frac{2s-1}{s^2(s+1)^3}\right\} = 5 - t - 5e^{-t} - 4e^{-t}\cdot t - 3e^{-t}\cdot\frac{1}{2}t^2 = 5 - t - 5e^{-t} - 4te^{-t} - \frac{3}{2}t^2e^{-t}.$

$$\frac{2s-1}{s^2(s+1)^3} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{(s+1)^2} + \frac{E}{(s+1)^3}$$

"Cover up" s^2: $\frac{2s-1}{(s+1)^3}\Big|_{s=0} = B \Rightarrow B=-1$; "Cover up" $(s+1)^3$: $\frac{2s-1}{s^2}\Big|_{s=-1} = E \Rightarrow E=-3$

$\Rightarrow \frac{A}{s} + \frac{C}{s+1} + \frac{D}{(s+1)^2} = \frac{2s-1}{s^2(s+1)^3} + \frac{1}{s^2} + \frac{3}{(s+1)^3} = \frac{2s-1+s^3+3s^2+3s+1+3s^2}{s^2(s+1)^3}$

$$= \frac{s^3+6s^2+5s}{s^2(s+1)^3} = \frac{s^2+6s+5}{s(s+1)^3} = \frac{(s+5)(s+1)}{s(s+1)^3} = \frac{s+5}{s(s+1)^2}$$

$\Rightarrow \frac{A}{s} + \frac{C}{s+1} + \frac{D}{(s+1)^2} = \frac{s+5}{s(s+1)^2}$

"Cover up" s: $A = \frac{s+5}{(s+1)^2}\Big|_{s=0} \Rightarrow A=5$

"Cover up" $(s+1)^2$: $D = \frac{s+5}{s}\Big|_{s=-1} \Rightarrow D=-4$

$\Rightarrow \frac{C}{s+1} = \frac{s+5}{s(s+1)^2} - \frac{5}{s} + \frac{4}{(s+1)^2}$

$$= \frac{s+\cancel{5}-5s^2-10s-\cancel{5}+4s}{s(s+1)^2} = \frac{-5s^2-5s}{s(s+1)^2}$$

$$= \frac{-5}{s+1} \Rightarrow C=-5$$

(23) $\mathcal{L}^{-1}\left\{\frac{s^3-2s^2-6s-6}{(s^2+2s+2)s}\right\}$ DNE $\quad \lim_{s\to\infty}\frac{s^3-2s^2-6s-6}{(s^2+2s+2)s}=1.$

(24) $\mathcal{L}^{-1}\left\{\frac{s^3-2s^2-6s-6}{(s^2+2s+2)s^2}\right\}=-3t+e^{-t}\cos t.$

$$\frac{s^3-2s^2-6s-6}{(s^2+2s+2)s^2}=\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+2s+2} \Rightarrow B=\left.\frac{s^3-2s^2-6s-6}{s^2+2s+2}\right|_{s=0}\Rightarrow B=-3$$

$$\frac{A}{s}+\frac{Cs+D}{s^2+2s+2}=\frac{s^3-2s^2-6s-6}{s^2(s^2+2s+2)}+\frac{3}{s^2}=\frac{s^3-2s^2\cancel{-6s}\cancel{-6}+3s^2\cancel{+6s}\cancel{+6}}{s^2(s^2+2s+2)}=\frac{s^3+s^2}{s^2(s^2+2s+2)}$$

$$\Rightarrow \frac{A}{s}+\frac{Cs+D}{s^2+2s+2}=\frac{s+1}{s^2+2s+2}\Rightarrow A=0;\ C=1\ D=1$$

$$\Rightarrow \mathcal{L}^{-1}\{F(s)\}=-3\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}}_{t}+\underbrace{\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+2s+2}\right\}}$$

$$\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\}=\mathcal{L}^{-1}\left\{\left.\frac{s}{s^2+1}\right|_{s\to s+1}\right\}=e^{-t}\cos t$$

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(25) $f(t)=\begin{cases}1, & t\in[0,1)\cup(2,\infty)\cup(1,2)\\ 3, & t=1\\ 4, & t=2\end{cases}$

$$\mathcal{L}\{f(t)\}=\int_0^\infty e^{-st}f(t)\,dt=\int_0^1 e^{-st}dt+\underbrace{\int_1^1 e^{-st}\cdot 3\,dt}_{0}+\int_1^2 e^{-st}dt+\underbrace{\int_2^2 e^{-st}\cdot 4\,dt}_{0}+\int_2^\infty e^{-st}dt$$

$$=\int_0^\infty e^{-st}dt$$

$$=\mathcal{L}\{1\}$$

$$=\frac{1}{s}$$

(26) $y' - 3y = e^{2t}$; $y(0) = 1$

$$\mathcal{L}\{y'\} - 3\mathcal{L}\{y\} = \mathcal{L}\{e^{2t}\}$$

$$sY(s) - y(0) - 3Y(s) = \frac{1}{s-2} \Rightarrow (s-3)Y(s) = \frac{1}{s-2} + 1 \Rightarrow Y(s) = \frac{s-1}{(s-2)(s-3)}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\left\{\frac{s-1}{(s-2)(s-3)}\right\} \Rightarrow \boxed{y(t) = -e^{2t} + 2e^{3t}}$$

$$\frac{s-1}{(s-2)(s-3)} = \frac{A}{s-2} + \frac{B}{s-3}$$

$$\left.\frac{s-1}{s-3}\right|_{s=2} = A \Rightarrow A = -1 \ ; \quad \left.\frac{s-1}{s-2}\right|_{s=3} = B \Rightarrow B = 2$$

(27) $y'' - 6y' + 9y = t^2 e^{3t}$; $y(0) = 2$, $y'(0) = 6$

$$\mathcal{L}\{y''\} - 6\mathcal{L}\{y'\} + 9Y(s) = \mathcal{L}\{t^2 e^{3t}\}$$

$$\mathcal{L}\{t^2\}\Big|_{s \to s-3} = \frac{2}{(s-3)^3}$$

$$s^2 Y(s) - s\,y(0) - y'(0) - 6(sY(s) - y(0)) + 9Y(s)$$

$$(s^2 - 6s + 9)Y(s) - 2s - 6 + 12$$

$$(s-3)^2 Y(s) - 2s + 6$$

$$(s-3)^2 Y(s) = \frac{2}{(s-3)^3} + 2s - 6 \Rightarrow Y(s) = \frac{2}{(s-3)^5} + \frac{2}{(s-3)}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\{Y(s)\} = 2\mathcal{L}^{-1}\left\{\frac{1}{s^5}\Big|_{s \to s-3}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{s}\Big|_{s \to s-3}\right\}$$

$$= 2e^{3t}\frac{1}{4!}t^4 + 2e^{3t}$$

$$\boxed{y(t) = \frac{1}{12}t^4 e^{3t} + 2e^{3t}}$$

(28) $y'' + 4y' + 6y = 1 + e^{-t}$; $y(0)=0$; $y'(0)=0$.

$$\mathcal{L}\{y''\} + 4\mathcal{L}\{y'\} + 6Y(s) = \mathcal{L}\{1\} + \mathcal{L}\{e^{-t}\}$$

$$s^2Y(s) - \underbrace{sy(0)}_{0} - \underbrace{y'(0)}_{0} + 4sY(s) - \underbrace{4y(0)}_{0} + 6Y(s) = \frac{1}{s} + \frac{1}{s+1}$$

$$(s^2+4s+6)Y(s) = \frac{1}{s} + \frac{1}{s+1} \Rightarrow \boxed{Y(s) = \frac{2s+1}{s(s+1)(s^2+4s+6)}}$$

$$\Rightarrow y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{6} + \frac{1}{3}e^{-t} - \frac{1}{2}e^{-2t}\cos(\sqrt{2}t) - \frac{\sqrt{2}}{3}e^{-2t}\sin(\sqrt{2}t).$$

$$\frac{2s+1}{s(s+1)(s^2+4s+6)} = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+4s+6}$$

$$\left.\frac{2s+1}{(s+1)(s^2+4s+6)}\right|_{s=0} = A \Rightarrow A = 1/6 \; ; \quad \left.\frac{2s+1}{s(s^2+4s+6)}\right|_{s=-1} = B \Rightarrow B = 1/3$$

$$\Rightarrow \frac{Cs+D}{s^2+4s+6} = \frac{2s+1}{s(s+1)(s^2+4s+6)} - \frac{1}{6s} - \frac{1}{3(s+1)}$$

$$= \frac{12s+6-(s+1)(s^2+4s+6)-2s(s^2+4s+6)}{6s(s+1)(s^2+4s+6)}$$

$$= \frac{-3s-10}{6(s^2+4s+6)}$$

$$-12s-6+(3s+1)(s^2+4s+6)$$

$$\cancel{-12s}\cancel{-6}+3s^3+12s^2+\cancel{18s}+s^2+4s\cancel{+6}$$

$$3s^3+13s^2+10s$$

$$s(3s^2+13s+10)$$

$$s(s+1)(3s+10)$$

$$\Rightarrow Cs+D = \frac{1}{2}s + \frac{5}{3} \Rightarrow C = -1/2 \quad D = -5/3$$

$$\mathcal{L}^{-1}\left\{\frac{s}{s^2+4s+6}\right\} = \mathcal{L}^{-1}\left\{\frac{s}{(s+2)^2+2}\right\} = e^{-2t}\cos(\sqrt{2}t) - \sqrt{2}e^{-2t}\sin(\sqrt{2}t)$$

$$\mathcal{L}^{-1}\left\{\frac{-1/2 s - 5/3}{s^2+4s+6}\right\} = -\frac{1}{2}e^{-2t}\cos(\sqrt{2}t) + \underbrace{\frac{1}{\sqrt{2}}e^{-2t}\sin(\sqrt{2}t) - \frac{5}{3\sqrt{2}}e^{-2t}\sin(\sqrt{2}t)}_{-\frac{\sqrt{2}}{3}e^{-2t}\sin(\sqrt{2}t)}$$

(29) $y'' - y' = e^t \cos t\ ;\ y(0) = y'(0) = 0$

$$s^2Y(s) - s\underbrace{y(0)}_{0} - \underbrace{y'(0)}_{0} - sY(s) + \underbrace{y(0)}_{0} = \mathcal{L}\{e^t\cos t\} = \mathcal{L}\{\cos t\}\Big|_{s\to s-1} = \frac{s-1}{(s-1)^2+1}$$

$$\Rightarrow (s^2 - s)Y(s) = \frac{s-1}{(s-1)^2+1} \Rightarrow Y(s) = \frac{1}{s((s-1)^2+1)} \Rightarrow \boxed{y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s(s^2-2s+2)}\right\}}$$

$$\frac{1}{s(s^2-2s+2)} = \frac{A}{s} + \frac{Bs+C}{s^2-2s+2} \Rightarrow \frac{1}{s^2-2s+2}\Big|_{s=0} = A \Rightarrow A = 1/2$$

$$\Rightarrow \frac{Bs+C}{s^2-2s+2} = \frac{1}{s(s^2-2s+2)} - \frac{1}{2s} = \frac{\cancel{2} - s^2 + 2s - \cancel{2}}{2s(s^2-2s+2)} = \frac{-s+2}{2(s^2-2s+2)}$$

$$\Rightarrow B = -1/2\ ;\ C = 1$$

$$\Rightarrow y(t) = \frac{1}{2} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{(s-1)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1}\right\}$$

$$= \frac{1}{2} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s-1+1}{(s-1)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2+1}\right\}$$

$$= \frac{1}{2} - \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\Big|_{s\to s-1}\right\} + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\Big|_{s\to s-1}\right\} = \boxed{\frac{1}{2} - \frac{1}{2}e^t\cos t + \frac{1}{2}e^t\sin t}$$