

$$\textcircled{1} \mathcal{L}\{te^{10t}\} = -\frac{d}{ds} \mathcal{L}\{e^{10t}\} = -\frac{d}{ds} \left(\frac{1}{s-10} \right) = \frac{1}{(s-10)^2} \quad (s > 10)$$

$$\textcircled{2} \mathcal{L}\{t^3 e^{-2t}\} = -\frac{d^3}{ds^3} \mathcal{L}\{e^{-2t}\} \quad \text{True, but it may be easier to use translation property}$$

$$= \mathcal{L}\{t^3\} \Big|_{s \rightarrow s+2}$$

$$= \frac{3!}{s^4} \Big|_{s \rightarrow s+2} = \frac{6}{(s+2)^4} \quad ; \quad (s > -2)$$

$$(s > 0) \mapsto (s+2 > 0)$$

$$\textcircled{3} \mathcal{L}\{e^t \sin(3t)\} = \mathcal{L}\{\sin(3t)\} \Big|_{s \rightarrow s-1} = \frac{3}{s^2+9} \Big|_{s \rightarrow s-1} = \frac{3}{(s-1)^2+9} \quad (s > 1)$$

$$(s > 0) \mapsto (s-1 > 0)$$

$$\textcircled{4} \mathcal{L}\{e^{-t} \sin^2 t\} = \mathcal{L}\{\sin^2 t\} \Big|_{s \rightarrow s+1} = \frac{2}{(s+1)((s+1)^2+4)} \quad (s > -1)$$

$$(s > 0) \mapsto (s+1 > 0)$$

$$\mathcal{L}\{\sin^2 t\} = \mathcal{L}\left\{ \frac{1-\cos(2t)}{2} \right\} = \mathcal{L}\left\{ \frac{1}{2} \right\} - \frac{1}{2} \mathcal{L}\{\cos(2t)\}$$

$$= \frac{1}{2s} - \frac{1}{2} \frac{s}{s^2+4} = \frac{2}{s(s^2+4)}$$

$$(s > 0) \quad (s > 0) \quad (s > 0)$$

$$\textcircled{5} \mathcal{L}\{t(e^t + e^{2t})^2\} = \mathcal{L}\{t(e^{2t} + 2e^{3t} + e^{4t})\}$$

$$= \mathcal{L}\{te^{2t}\} + 2\mathcal{L}\{te^{3t}\} + \mathcal{L}\{te^{4t}\}$$

$$= \mathcal{L}\{t\} \Big|_{s \rightarrow s-2} + 2\mathcal{L}\{t\} \Big|_{s \rightarrow s-3} + \mathcal{L}\{t\} \Big|_{s \rightarrow s-4}$$

$$= \boxed{\frac{1}{(s-2)^2} + \frac{2}{(s-3)^2} + \frac{1}{(s-4)^2}} \quad (s > 4)$$

$$(s > 2) \cap (s > 3) \cap (s > 4)$$

AMPA

$$\begin{aligned} \textcircled{6} \mathcal{L}\{t \cos(2t)\} &= -\frac{d}{ds} \mathcal{L}\{\cos(2t)\} = -\frac{d}{ds} \frac{s}{s^2+4} = \\ &= -\frac{s^2+4-2s^2}{(s^2+4)^2} = \boxed{\frac{s^2-4}{(s^2+4)^2}} \quad (s>0) \end{aligned}$$

$$\begin{aligned} \textcircled{7} \mathcal{L}\{e^{5t} \sinh(3t)\} &= \mathcal{L}\{\sinh(3t)\} \Big|_{s \rightarrow s-5} = \frac{3}{s^2-9} \Big|_{s \rightarrow s-5} \\ &= \boxed{\frac{3}{(s-5)^2-9}} \quad (s>8) \quad \begin{array}{l} s>3 \\ s-5>3 \quad s>8 \end{array} \end{aligned}$$

$$\begin{aligned} \textcircled{8} \mathcal{L}\{te^{2t} \sin(6t)\} &= -\frac{d}{ds} \mathcal{L}\{e^{2t} \sin(6t)\} \\ &= -\frac{d}{ds} \mathcal{L}\{\sin(6t)\} \Big|_{s \rightarrow s-2} \\ &= -\frac{d}{ds} \frac{6}{s^2+36} \Big|_{s \rightarrow s-2} = -\frac{d}{ds} \frac{6}{(s-2)^2+36} \\ & \quad (s>0) \mapsto (s-2>0) \\ &= \boxed{\frac{12(s-2)}{((s-2)^2+36)^2}} \quad (s>2) \end{aligned}$$

$$\begin{aligned} \textcircled{9} \mathcal{L}\{e^{-2t}(t^3+1)^2\} &= \mathcal{L}\{(t^3+1)^2\} \Big|_{s \rightarrow s+2} \\ &= \mathcal{L}\{t^6+2t^3+1\} \Big|_{s \rightarrow s+2} \\ &= \left(\frac{6!}{s^7} + \frac{2 \cdot 3!}{s^4} + \frac{1}{s} \right) \Big|_{s \rightarrow s+2} \quad (s>0) \mapsto (s+2>0) \\ &= \boxed{\frac{6!}{(s+2)^7} + \frac{12}{(s+2)^4} + \frac{1}{s+2}} \quad (s>-2) \end{aligned}$$

$$\begin{aligned} \textcircled{10} \mathcal{L}\{te^{at} \sin(bt)\} &= -\frac{d}{ds} \mathcal{L}\{e^{at} \sin(bt)\} = -\frac{d}{ds} \mathcal{L}\{\sin(bt)\} \Big|_{s \rightarrow s-a} \\ &= -\frac{d}{ds} \frac{b}{s^2+b^2} \Big|_{s \rightarrow s-a} = -\frac{d}{ds} \frac{b}{(s-a)^2+b^2} \\ & \quad \begin{array}{l} s>0 \\ \downarrow \\ s-a>0 \end{array} \\ &= \boxed{\frac{2b(s-a)}{((s-a)^2+b^2)^2}} \quad (s>a) \end{aligned}$$

$$(11) \quad y'' - 2y' + y = 0; \quad y(0) = 2; \quad y'(0) = 3.$$

$$\mathcal{L}\{y(t)\} = ? \quad (Y(s) = ?)$$

$$\mathcal{L}\{y'' - 2y' + y\} = \mathcal{L}\{0\}$$

$$\underbrace{\mathcal{L}\{y''\}}_{s^2 Y(s) - sy(0) - y'(0)} - 2 \underbrace{\mathcal{L}\{y'\}}_{sY(s) - y(0)} + \underbrace{\mathcal{L}\{y\}}_{Y(s)} = 0$$

$$s^2 Y(s) - 2s - 3 - 2s Y(s) + 4 + Y(s) = 0$$

$$(s^2 - 2s + 1) Y(s) = 2s - 1 \Rightarrow$$

$$Y(s) = \frac{2s - 1}{s^2 - 2s + 1}$$