(1) $y'' + 3y' + 2y = \frac{1}{1+e^x}$

y_c : $m^2 + 3m + 2 = 0 \Rightarrow (m+1)(m+2) = 0 \Rightarrow \boxed{y_c = c_1 e^{-x} + c_2 e^{-2x}}$

y_p : $W = \begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix} = -e^{-3x}$

$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \frac{1}{1+e^x} & -2e^{-2x} \end{vmatrix} = -\frac{e^{-2x}}{1+e^x}$; $W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \frac{1}{1+e^x} \end{vmatrix} = \frac{e^{-x}}{1+e^x}$

$u_1' = \frac{e^x}{1+e^x} \Rightarrow \boxed{u_1 = \ln(1+e^x)}$

$u_2' = -\frac{e^{2x}}{1+e^x} = -\frac{e^x(e^x+1-1)}{e^x+1} = -e^x + \frac{e^x}{e^{x+1}} \Rightarrow \boxed{u_2 = -e^x + \ln(e^x+1)}$

$y_p = u_1 e^{-x} + u_2 e^{-2x} = e^{-x}\ln(1+e^x) - e^{-x} + e^{-2x}\ln(e^x+1)$

$\boxed{y_p = -e^{-x} + (e^{-x} + e^{-2x})\ln(e^x+1)}$

y $\boxed{y = c_1 e^{-x} + c_2 e^{-2x} + (e^{-x}+e^{-2x})\ln(1+e^x)}$ $x \in \mathbb{R}$

(2) $x^2y'' - 3xy' - 2y = 0$

Cauchy-Euler w/ $a=1, b=-3, c=-2$

Char. Eqn. : $m^2 - 4m - 2 = 0 \Rightarrow m = \frac{4 \pm 2\sqrt{6}}{2} = 2 \pm \sqrt{6}$

$\boxed{y = c_1 x^{2+\sqrt{6}} + c_2 x^{2-\sqrt{6}}}$ $x \in (0, \infty)$

(3) $xy'' + y' = 0$ Make it Cauchy-Euler : multiply by x

$x^2y'' + xy' = 0$ Cauchy-Euler w/ $a=1, b=1, c=0$

Char. Eqn. : $m^2 = 0 \Rightarrow$ Double root $\boxed{m=0}$

OR: Substitution $w = y'$
$\Rightarrow xw' + w = 0 \Rightarrow$ Separable / Linear
Solve for w
Solve for y

$y = c_1 x^0 + c_2 x^0 \ln x$

$\boxed{y = c_1 + c_2 \ln x}$ $x \in (0, \infty)$

(4) $x^2y'' - xy' + y = \ln x$

Complementary Solution $\quad x^2y'' - xy' + y = 0 \quad$ Cauchy-Euler

$a = 1, \ b = -1, \ c = 1$

Char. Eq. : $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0 \Rightarrow$ double root $m = 1$

$$\boxed{y_c = c_1 x + c_2 x \ln x}$$

Particular Solution : Standard Form : $y'' - \frac{1}{x}y' + \frac{1}{x^2}y = \frac{\ln x}{x^2}$

$$W = \begin{vmatrix} x & x\ln x \\ 1 & 1+\ln x \end{vmatrix} = x + x\ln x - x\ln x = x$$

$$W_1 = \begin{vmatrix} 0 & x\ln x \\ \frac{\ln x}{x^2} & 1+\ln x \end{vmatrix} = -\frac{\ln^2 x}{x}$$

$$W_2 = \begin{vmatrix} x & 0 \\ 1 & \frac{\ln x}{x^2} \end{vmatrix} = \frac{\ln x}{x}$$

$$u_1' = -\frac{1}{x^2}\ln^2 x \Rightarrow u_1 = \int -\frac{1}{x^2}\ln^2 x\,dx = \int \left(\frac{1}{x}\right)' \ln^2 x\,dx$$

$$= \frac{1}{x}\ln^2 x - \int \frac{1}{x}\cdot 2\ln x \cdot \frac{1}{x}$$

$$= \frac{1}{x}\ln^2 x + 2\int -\frac{1}{x^2}\ln x\,dx$$

$$= \frac{1}{x}\ln^2 x + 2\int \left(\frac{1}{x}\right)' \ln x\,dx$$

$$= \frac{1}{x}\ln^2 x + \frac{2}{x}\ln x - 2\int \frac{1}{x^2}\,dx$$

$$= \frac{1}{x}\ln^2 x + \frac{2}{x}\ln x + \frac{2}{x}$$

$$\boxed{u_1 = \frac{1}{x}\ln^2 x + \frac{2}{x}\ln x + \frac{2}{x}}$$

$$u_2' = \frac{\ln x}{x^2} \Rightarrow u_2 = \int \frac{1}{x^2} \ln x = -\int \left(\frac{1}{x}\right)' \ln x \, dx$$
$$= -\frac{1}{x} \ln x + \int \frac{1}{x^2} dx$$
$$= -\frac{1}{x} \ln x - \frac{1}{x}$$

$$\boxed{u_2 = -\frac{1}{x} \ln x - \frac{1}{x}}$$

$$\Rightarrow y_p = u_1 x + u_2 x \ln x = \ln^2 x + 2\ln x + 2 - \ln^2 x - \ln x = \underline{\underline{2 + \ln x}}$$

$$\boxed{y_p = 2 + \ln x}$$

General Sol. $\boxed{y = c_1 x + c_2 x \ln x + 2 + \ln x}$ $\quad x \in (0, \infty)$

5 $y'' + 3y' + 2y = \sin(e^x)$

(y_c): $m^2 + 3m + 2 = 0 \Rightarrow (m+1)(m+2) = 0 \Rightarrow \boxed{y_c = c_1 e^{-x} + c_2 e^{-2x}}$

(y_p): $W = -e^{-3x}$ (same as problem #1)

$$W_1 = \begin{vmatrix} 0 & e^{-2x} \\ \sin(e^x) & -2e^{-2x} \end{vmatrix} = -e^{-2x} \sin(e^x)$$

$$W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & \sin(e^x) \end{vmatrix} = e^{-x} \sin(e^x)$$

$$u_1' = e^x \sin(e^x) \Rightarrow \boxed{u_1 = -\cos(e^x)}$$

$$u_2' = -e^{2x} \sin(e^x) \Rightarrow u_2 = \int -e^x \cdot e^x \sin(e^x) dx = \int e^x (\cos(e^x))' dx$$
$$= e^x \cos(e^x) - \int e^x \cos(e^x)$$
$$= \boxed{e^x \cos(e^x) - \sin(e^x)}$$

$$\Rightarrow y_p = u_1 e^{-x} + u_2 e^{-2x}$$
$$= \cancel{-e^{-x} \cos(e^x)} + \cancel{e^{-x} \cos(e^x)} - e^{-2x} \sin(e^x) \Rightarrow \boxed{y_p = -e^{-2x} \sin(e^x)}$$

$\boxed{y = c_1 e^{-x} + c_2 e^{-2x} - e^{-2x} \sin(e^x)}$ $\quad x \in \mathbb{R}$

(6) $y'' + 2y' + y = e^{-x}\ln x$

(y_c) $m^2 + 2m + 1 = 0 \Rightarrow \boxed{y_c = c_1 e^{-x} + c_2 x e^{-x}}$

(y_p)

$$W = \begin{vmatrix} e^{-x} & xe^{-x} \\ -e^{-x} & (1-x)e^{-x} \end{vmatrix} = e^{-2x}$$

$$W_1 = \begin{vmatrix} 0 & xe^{-x} \\ e^{-x}\ln x & (1-x)e^{-x} \end{vmatrix} = -xe^{-2x}\ln x \Rightarrow u_1' = -x\ln x$$

$$\Rightarrow \boxed{u_1 = -\frac{x^2}{2}\ln x + \frac{x^2}{4}}$$

$$W_2 = \begin{vmatrix} e^{-x} & 0 \\ -e^{-x} & e^{-x}\ln x \end{vmatrix} = e^{-2x}\ln x$$

$$\Rightarrow u_2' = \ln x \Rightarrow \boxed{u_2 = x\ln x - x}$$

$$y_p = \left(-\frac{x^2}{2}\ln x + \frac{x^2}{4}\right)e^{-x} + \left(x^2\ln x - x^2\right)e^{-x}$$

$$= \boxed{\left(\frac{x^2}{2}\ln x - \frac{3x^2}{4}\right)e^{-x}}$$

$$\boxed{y = c_1 e^{-x} + c_2 x e^{-x} + \left(\frac{1}{2}\ln x - \frac{3}{4}\right)x^2 e^{-x}}$$

$x \in (0, \infty)$

(7) $3x^2y'' + 6xy' + y = 0$ Cauchy-Euler w/ $a=3$, $b=6$, $c=1$

Char. Eq. : $3m^2 + 3m + 1 = 0$

$\Delta = 9 - 12 = -3 \Rightarrow m = \frac{-3 \pm i\sqrt{3}}{6} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{6}$

$$\boxed{y = \frac{1}{\sqrt{x}}\left(c_1\cos\left(\frac{\sqrt{3}}{6}\ln x\right) + c_2\sin\left(\frac{\sqrt{3}}{6}\ln x\right)\right).}$$

$x \in (0, \infty)$

(8) $x^2y'' + 5xy' + 4y = 0$ Cauchy-Euler w/ $a=1$, $b=5$, $c=4$

Char. Eqn.: $m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$$\boxed{y = c_1 x^{-2} + c_2 x^{-2}\ln x}$$

$x \in (0, \infty)$

(9) $x^2y''-xy'+y=4x\ln x \rightsquigarrow y''-\frac{1}{x}y'+\frac{1}{x^2}y=\frac{4\ln x}{x}$

(y_c) : $x^2y''-xy'+y=0$ Cauchy-Euler w/ $a=1, b=-1, c=1$

Char. Eqn.: $m^2-2m+1=0 \Rightarrow (m-1)^2=0$

$$\boxed{y_c=C_1x+C_2x\ln x}$$

(y_p) : $W=\begin{vmatrix} x & x\ln x \\ 1 & \ln x+1 \end{vmatrix}=x$

$W_1=\begin{vmatrix} \cancel{4\,0\,x} & x\ln x \\ \frac{4\ln x}{x} & \ln x+1 \end{vmatrix}=-4\ln^2 x$; $W_2=\begin{vmatrix} x & 0 \\ 1 & \frac{4\ln x}{x} \end{vmatrix}=4\ln x$

$u_1'=-\frac{4}{x}\ln^2 x \Rightarrow u_1=\int-\frac{4}{x}\ln^2 x\,dx=-\frac{4}{3}\ln^3 x$

$u_2'=\frac{4\ln x}{x} \Rightarrow u_2=\int\frac{4}{x}\ln x\,dx=2\ln^2 x$

$y_p=u_1x+u_2x\ln x=-\frac{4}{3}x\ln^3 x+2x\ln^3 x=\boxed{\frac{2}{3}x\ln^3 x}$

$$\boxed{y=c_1x+c_2x\ln x+\frac{2}{3}x\ln^3 x}$$ $x\in(0,\infty)$

(10) $x^2y''-xy'+2y=0$ Cauchy-Euler w/ $a=1, b=-1, c=2$

Char. Eqn.: $m^2-2m+2=0$

$\Delta=-4 \Rightarrow m=\frac{2\pm 2i}{2}=1\pm i$

$$\boxed{y=x\left(c_1\cos(\ln x)+c_2\sin(\ln x)\right)}$$ $x\in(0,\infty)$

(11) $y'' - 2y' + y = \frac{e^x}{x}$

(y_c) : $m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0$ $\quad \boxed{y_c = c_1 e^x + c_2 x e^x}$

(y_p) :

$$W = \begin{vmatrix} e^x & xe^x \\ e^x & (x+1)e^x \end{vmatrix} = e^{2x}$$

$$W_1 = \begin{vmatrix} 0 & xe^x \\ \frac{1}{x}e^x & (x+1)e^x \end{vmatrix} = -e^{2x} \Rightarrow u_1' = -1 \Rightarrow \boxed{u_1 = -x}$$

$$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{1}{x}e^x \end{vmatrix} = \frac{1}{x}e^{2x} \Rightarrow u_2' = \frac{1}{x} \Rightarrow \boxed{u_2 = \ln x}$$

$$\boxed{y_p = \underbrace{-xe^x}_{\text{gets absorbed by } c_2} + xe^x \ln x} \qquad \boxed{y = c_1 e^x + c_2 x e^x + xe^x \ln x}$$

$x \in (0, \infty)$

(12) $y'' + 4y = \sin^2(2x)$ $\qquad y_c = c_1 \cos(2x) + c_2 \sin(2x).$

$$W = \begin{vmatrix} \cos(2x) & \sin(2x) \\ -2\sin(2x) & 2\cos(2x) \end{vmatrix} = 2$$

$$W_1 = \begin{vmatrix} 0 & \sin(2x) \\ \sin^2(2x) & 2\cos(2x) \end{vmatrix} = -\sin^3(2x) \Rightarrow u_1' = -\frac{1}{2}\sin^3(2x)$$

$$W_2 = \begin{vmatrix} \cos(2x) & 0 \\ -2\sin(2x) & \sin^2(2x) \end{vmatrix} = \sin^2(2x)\cos(2x)$$

$$\Rightarrow u_2' = \frac{1}{2}\sin^2(2x)\cos(2x) \Rightarrow u_2 = \frac{1}{4} \cdot \frac{1}{3}\sin^3(2x)$$

$$\boxed{u_2 = \frac{1}{12}\sin^3(2x)}$$

$$u_1 = -\frac{1}{2}\int \operatorname{sen}^3(2x)\,dx = -\frac{1}{2}\int \operatorname{sen}^2(2x)\operatorname{sen}(2x)\,dx$$

$$= -\frac{1}{2}\int \left(1-\cos^2(2x)\right)\operatorname{sen}(2x)\,dx$$

$$= -\frac{1}{2}\left(\int \operatorname{sen}(2x)\,dx + \int -\cos^2(2x)\operatorname{sen}(2x)\,dx\right)$$

$$= -\frac{1}{2}\left(-\frac{1}{2}\cos(2x) + \frac{1}{6}\cos^3(2x)\right) = \boxed{\frac{1}{4}\cos(2x) - \frac{1}{12}\cos^3(2x)}$$

$$y_p = \frac{1}{4}\cos^2(2x) - \frac{1}{12}\cos^4(2x) + \frac{1}{12}\operatorname{sen}^4(2x)$$

$$= \frac{1}{4}\cos^2(2x) + \frac{1}{12}\left(\operatorname{sen}^4(2x) - \cos^4(2x)\right)$$

$$= \frac{1}{4}\cos^2(2x) + \frac{1}{12}\left(\operatorname{sen}^2(2x) - \cos^2(2x)\right)\underbrace{\left(\operatorname{sen}^2(2x) + \cos^2(2x)\right)}_{1}$$

$$= \boxed{\frac{1}{12}\operatorname{sen}^2(2x) + \frac{1}{6}\cos^2(2x)}$$

$$\Rightarrow \boxed{y = c_1\cos(2x) + c_2\operatorname{sen}(2x) + \frac{1}{12}\operatorname{sen}^2(2x) + \frac{1}{6}\cos^2(2x)} \qquad x \in \mathbb{R}$$

(13) $x^2y'' - xy' = x^3e^x$ $\qquad$ $y'' - y'\frac{1}{x} = xe^x$

(y_c) : $x^2y'' - xy' = 0$ Cauchy-Euler w/ $a=1, b=-1, c=0$

Char. Eqn.: $m^2 - 2m = 0 \Rightarrow m = 0, 2$

$$\boxed{y_c = c_1 + c_2x^2}$$

(y_p)

$$W = \begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix} = 2x$$

$$W_1 = \begin{vmatrix} 0 & x^2 \\ xe^x & 2x \end{vmatrix} = -x^3e^x \Rightarrow u_1' = -\frac{x^2}{2}e^x \Rightarrow \boxed{u_1 = -\frac{1}{2}x^2e^x + xe^x - e^x}$$

$$W_2 = \begin{vmatrix} 1 & 0 \\ 0 & xe^x \end{vmatrix} = xe^x \Rightarrow u_2' = \frac{1}{2}e^x \Rightarrow \boxed{u_2 = \frac{1}{2}e^x}$$

$$y_p = \cancel{-\frac{1}{2}x^2e^x} + xe^x - e^x + \cancel{\frac{1}{2}x^2e^x} \qquad \boxed{y_p = xe^x - e^x}$$

$$\boxed{y = c_1 + c_2x^2 + (xe^x - e^x)}$$ $x \in \mathbb{R}$

(14) $25x^2y'' + 25xy' + y = 0$ $\qquad$ Cauchy-Euler w/ $a=b=25; c=1$

Char. Eqn. : $25m^2 + 1 = 0$

$m = \pm\frac{1}{5}i$

$$\boxed{y = c_1\cos\left(\frac{1}{5}\ln x\right) + c_2\sin\left(\frac{1}{5}\ln x\right)}$$ $x \in (0, \infty)$

15) $y''' - 3y'' + 3y' - y = \frac{e^x}{x}$

Char. Eqn. : $m^3 - 3m^2 + 3m - 1 = 0 \Rightarrow (m-1)^3 = 0$

$$\boxed{y_c = c_1 e^x + c_2 x e^x + c_3 x^2 e^x}$$

Particular Sol. :

$$W = \begin{vmatrix} e^x & xe^x & x^2e^x \\ e^x & (x+1)e^x & (x^2+2x)e^x \\ e^x & (x+2)e^x & (x^2+4x+2)e^x \end{vmatrix} = e^{3x} \begin{vmatrix} 1 & x & x^2 \\ 1 & x+1 & x^2+2x \\ 1 & x+2 & x^2+4x+2 \end{vmatrix}$$

$$= e^{3x} \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 2 & 4x+2 \end{vmatrix} = e^{3x}(4x+2-4x) = 2e^{3x}$$

$$W_1 = \begin{vmatrix} 0 & xe^x & x^2e^x \\ 0 & (x+1)e^x & (x^2+2x)e^x \\ \frac{1}{x}e^x & (x+2)e^x & (x^2+4x+2)e^x \end{vmatrix} = \frac{1}{x}e^{3x}\left(x^3+2x^2-x^3-x^2\right) = xe^{3x}$$

$$u_1' = \frac{x}{2} \quad \boxed{u_1 = \frac{x^2}{4}}$$

$$W_2 = \begin{vmatrix} e^x & 0 & x^2e^x \\ e^x & 0 & (x^2+2x)e^x \\ e^x & \frac{1}{x}e^x & (x^2+4x+2)e^x \end{vmatrix} = -\frac{1}{x}e^{3x}\cdot 2x = -2e^{3x} \Rightarrow u_2' = -1$$

$$\boxed{u_2 = -x}$$

$$W_3 = \begin{vmatrix} e^x & xe^x & 0 \\ e^x & (x+1)e^x & 0 \\ e^x & (x+2)e^x & \frac{1}{x}e^x \end{vmatrix} = \frac{1}{x}e^{3x}$$

$$\Rightarrow u_3' = \frac{1}{2x} \Rightarrow \boxed{u_3 = \frac{1}{2}\ln x}$$

$$y_p = \underbrace{\frac{x^2}{4}e^x - x^2e^x}_{\text{gets absorbed by } c_3} + \frac{1}{2}\ln x\,(x^2e^x)$$

$$\boxed{y = c_1e^x + c_2xe^x + c_3x^2e^x + \frac{1}{2}x^2e^x\ln x}$$

$x \in (0, \infty)$

(16) $y''' - 3y'' + 2y' = \dfrac{e^{3x}}{1+e^x}$

Comp. Sol. : $m^3 - 3m^2 + 2m = 0$

$m(m-1)(m-2) = 0$ $\qquad y_c = c_1 + c_2 e^x + c_3 e^{2x}$

y_p

$$W = \begin{vmatrix} 1 & e^x & e^{2x} \\ 0 & e^x & 2e^{2x} \\ 0 & e^x & 4e^{2x} \end{vmatrix} = 2e^{3x}$$

$$W_1 = \begin{vmatrix} 0 & e^x & e^{2x} \\ 0 & e^x & 2e^{2x} \\ \frac{e^{3x}}{1+e^x} & e^x & 4e^{2x} \end{vmatrix} = \frac{e^{3x}}{1+e^x} \cdot e^{3x} \Rightarrow u_1' = \frac{1}{2} \frac{e^{3x}}{1+e^x}$$

$$W_2 = \begin{vmatrix} 1 & 0 & e^{2x} \\ 0 & 0 & 2e^{2x} \\ 0 & \frac{e^{3x}}{1+e^x} & 4e^{2x} \end{vmatrix} = -\frac{2e^{5x}}{1+e^x} \Rightarrow u_2' = -\frac{e^{2x}}{1+e^x}$$

$$W_3 = \begin{vmatrix} 1 & e^x & 0 \\ 0 & e^x & 0 \\ 0 & e^x & \frac{e^{3x}}{1+e^x} \end{vmatrix} = \frac{e^{4x}}{1+e^x} \Rightarrow u_3' = \frac{1}{2} \frac{e^x}{1+e^x}$$

$$u_1 = \frac{1}{2}\int \frac{e^{3x}}{1+e^x}dx = \frac{1}{2}\int e^x \cdot \frac{e^{2x}}{1+e^x}dx = \frac{1}{2}\int e^x \cdot \left(\frac{(e^{2x}-1)+1}{1+e^x}\right)dx$$

$$= \frac{1}{2}\int e^x\left(\frac{(e^x-1)\cancel{(e^x+1)}}{\cancel{e^x+1}} + \frac{1}{1+e^x}\right)dx$$

$$= \frac{1}{2}\int\left(e^{2x} - e^x + \frac{e^x}{1+e^x}\right)dx = \frac{1}{2}\left(\frac{1}{2}e^{2x} - e^x + \ln(1+e^x)\right)$$

$$u_2 = -\int \frac{e^{2x}}{1+e^x} = \boxed{-e^x + \ln(e^x+1)}$$ (See Problem 1)

$$u_3 = \frac{1}{2}\ln(e^x+1)$$

$$y_p = \underbrace{\frac{1}{4}e^{2x}}\underbrace{-\frac{1}{2}e^x} + \frac{1}{2}\ln(1+e^x) \underbrace{- e^{2x}} + e^x\ln(e^x+1) + \frac{1}{2}e^{2x}\ln(e^x+1)$$

$x \in \mathbb{R}$ $\qquad y = c_1 + c_2 e^x + c_3 e^{2x} + \left(\frac{1}{2}e^{2x} + e^x + \frac{1}{2}\right)\ln(e^x+1)$

(17) $x^2y'' + xy' + y = \sec(\ln x) \rightsquigarrow y'' + \frac{1}{x}y' + \frac{1}{x^2}y = \frac{1}{x^2}\sec(\ln x)$

(y_c) $x^2y'' + xy' + y = 0$ Cauchy-Euler w/ $a = b = c = 1$

Char. Eq.: $m^2 + 1 = 0 \Rightarrow m = \pm i$

$$y_c = c_1\cos(\ln x) + c_2\sin(\ln x)$$

(y_p)

$$W = \begin{vmatrix} \cos(\ln x) & \sin(\ln x) \\ -\frac{1}{x}\sin(\ln x) & \frac{1}{x}\cos(\ln x) \end{vmatrix} = \frac{1}{x}$$

$$W_1 = \begin{vmatrix} 0 & \sin(\ln x) \\ \frac{1}{x^2}\sec(\ln x) & \frac{1}{x}\cos(\ln x) \end{vmatrix} = -\frac{1}{x^2}\tan(\ln x)$$

$$W_2 = \begin{vmatrix} \cos(\ln x) & 0 \\ -\frac{1}{x}\sin(\ln x) & \frac{1}{x^2}\sec(\ln x) \end{vmatrix} = \frac{1}{x^2}$$

$$\Rightarrow u_1' = -\frac{1}{x}\tan(\ln x) \Rightarrow \boxed{u_1 = \ln|\cos(\ln x)|}$$

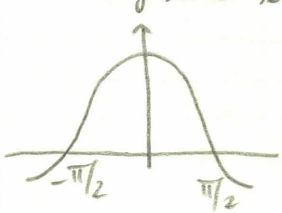
$$\Rightarrow u_2' = \frac{1}{x} \Rightarrow \boxed{u_2 = \ln x}$$

$$y_p = u_1\cos(\ln x) + u_2\sin(\ln x) = \boxed{\cos(\ln x)\ln|\cos(\ln x)| + \ln x\sin(\ln x)}$$

An interval of validity?

$x \in (0, \infty)$ is forced by the $\ln x$ in the original equation

But the solution also requires $\boxed{\cos(\ln x) > 0}$ (also takes care of the $\sec(\ln x)$ in the original problem).



Pick an interval where cos is positive: $(-\frac{\pi}{2}, \frac{\pi}{2})$

$$\Rightarrow \ln x \in (-\frac{\pi}{2}, \frac{\pi}{2}) \Rightarrow \boxed{x \in (e^{-\pi/2}, e^{\pi/2})}$$