Math 2552 - Differential Equations **Homework 1**

Sections F1 – F4; L1 – L4

Georgia Institute of Technology, Fall 2015

Part I - Separable Equations

Solve the following differential equations using separation of variables:

1. $\dfrac{dy}{dx} = \sin(5x)$

2. $xy' = 4y$

3. $(4y + yx^2)\, dy - (2x + xy^2)\, dx = 0$

4. $\dfrac{dy}{dx} = \dfrac{xy + 3x - y - 3}{xy - 2x + 4y - 8}$

5. $y\, dy = 4x\sqrt{y^2+1}\, dx;\;\; y(0) = 1$

6. $\dfrac{dx}{dy} = 4(x^2+1);\;\; x\left(\dfrac{\pi}{4}\right) = 1$

7. $x^2y' = y - xy;\;\; y(-1) = -1$

8. $e^y \sin(2x)\, dx + \cos x(e^{2y} - y)\, dy = 0.$

Differential equations of the form $y'(x) = F(ax+by+c)$, with $b \neq 0$, can be reduced to a separable equation by making the substitution:

$$u = ax + by + c.$$

Differentiating both sides above, we have:

$$\frac{du}{dx} = a + b\frac{dy}{dx} \;\;\Rightarrow\;\; \frac{dy}{dx} = \frac{1}{b}\left(\frac{du}{dx} - a\right),$$

and the original equation may be rewritten as:

$$\frac{du}{dx} = a + bF(u).$$

Use this method to solve the equations:

9. $\dfrac{dy}{dx} = (x+y+1)^2$

10. $\dfrac{dy}{dx} = 1 + e^{y-x+5}$

11. $\dfrac{dy}{dx} = 2 + \sqrt{y - 2x + 3}.$

Part II - First Order Linear DEs; Integrating Factors

Solve the following first order linear differential equations, and state an interval where your solution is valid.

1. $x\dfrac{dy}{dx} - 4y = x^6 e^x$

2. $\dfrac{dy}{dx} + 2xy = x;\;\; y(0) = -3$

3. $y' + 3x^2 y = x^2$

4. $\dfrac{dy}{dx} + 5y = 20;\;\; y(0) = 2$

5. $x\,dy = (x\sin x - y)\,dx$

6. $(1+e^x)\dfrac{dy}{dx} + e^x y = 0$

7. $x\dfrac{dy}{dx} + 4y = x^3 - x$

8. $(x+1)\dfrac{dy}{dx} + y = \ln x;\;\; y(1) = 10$

9. $(x+2)^2\dfrac{dy}{dx} = 5 - 8y - 4xy$

10. $\dfrac{dy}{dx} + y = \dfrac{1-e^{-2x}}{e^x + e^{-x}}$

11. $x^2 y' + x(x+2)y = e^x$

12. $x\dfrac{dy}{dx} + (3x+1)y = e^{-3x}$

13. $x(x-2)y' + 2y = 0;\;\; y(3) = 6$

14. $\dfrac{dy}{dx} = \dfrac{y}{y-x};\;\; y(5) = 2$

15. $(x+4y^2)\,dy + 2y\,dx = 0$

16. $y\,dx + (xy + 2x - ye^y)\,dy = 0.$