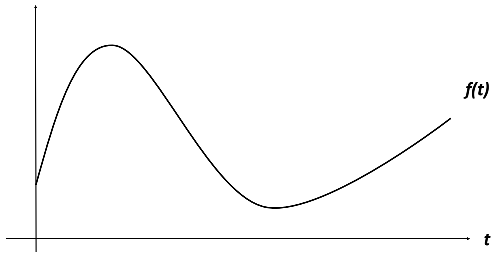
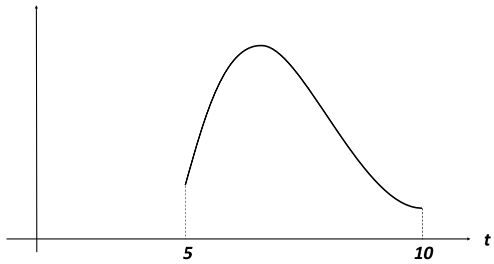
NAME: .... Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 9

1. In the picture to the left below there is the graph of a function f. To the right is the graph obtained by shifting the graph of f by 5 units to the right, then "turning off" the result at $t = 10$. Assuming the function to the right is 0 everywhere except on the interval $[5, 10]$, express it in terms of f and unit step functions.





2. Find:

$$\mathscr{L}^{-1}\left\{\frac{se^{-4s}}{s^2+9}\right\}$$

3. Find:

$$\mathscr{L}\left\{e^{t-2}\big(u_2(t)-u_5(t)\big)\right\}$$

Laplace transforms of some basic functions

$\mathscr{L}\{1\} = \frac{1}{s};\ s > 0$

$\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}};\ s > 0$

$\mathscr{L}\{e^{kt}\} = \frac{1}{s-k};\ s > k$

$\mathscr{L}\{\sin(kt)\} = \frac{k}{s^2+k^2};\ s > 0$

$\mathscr{L}\{\cos(kt)\} = \frac{s}{s^2+k^2};\ s > 0$

$\mathscr{L}\{u_a(t)\} = \frac{e^{-as}}{s};\ s > 0$

$\mathscr{L}\{\sinh(kt)\} = \frac{k}{s^2-k^2};\ s > |k|$

$\mathscr{L}\{\cosh(kt)\} = \frac{s}{s^2-k^2};\ s > |k|$

Inverse Laplace transforms of some basic functions

$\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$\mathscr{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{1}{(n-1)!}t^{n-1}$

$\mathscr{L}^{-1}\left\{\frac{1}{s-k}\right\} = e^{kt}$

$\mathscr{L}^{-1}\left\{\frac{1}{s^2+k^2}\right\} = \frac{1}{k}\sin(kt)$

$\mathscr{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos(kt)$

$\mathscr{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} = u_a(t)$

$\mathscr{L}^{-1}\left\{\frac{1}{s^2-k^2}\right\} = \frac{1}{k}\sinh(kt)$

$\mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} = \cosh(kt)$

Properties of the Laplace and Inverse Laplace transform

Translation Theorem I:

$$\mathscr{L}\left\{e^{kt}f(t)\right\} = F(s-k) = \mathscr{L}\{f(t)\}\big|_{s\to s-k}$$

$$\mathscr{L}^{-1}\{F(s-k)\} = \mathscr{L}^{-1}\left\{F(s)\big|_{s\to s-k}\right\} = e^{kt}\mathscr{L}^{-1}\{F(s)\} = e^{kt}f(t)$$

Translation Theorem II:

$$\mathscr{L}\left\{f(t-a)u_a(t)\right\} = e^{-as}F(s) = e^{-as}\mathscr{L}\{f(t)\}$$

$$\mathscr{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u_a(t) = \mathscr{L}^{-1}\{F(s)\}\big|_{t\to t-a}\,u_a(t)$$

Derivatives of Laplace Transforms:

$$\mathscr{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n}F(s)$$

$$\mathscr{L}^{-1}\left\{F^{(n)}(s)\right\} = (-1)^n t^n f(t)$$

Laplace Transform of Periodic Functions:
If $f(t)$ is piecewise continuous on $[0,\infty)$, of exponential order, and periodic with period T:

$$\mathscr{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt$$

Laplace Transforms of Derivatives:

$$\begin{aligned}
\mathscr{L}\{y'\} &= sY(s) - y(0)\\
\mathscr{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0)\\
\mathscr{L}\{y'''\} &= s^3Y(s) - s^2y(0) - sy'(0) - y''(0)\\
&\ \vdots\\
\mathscr{L}\{y^{(n)}(t)\} &= s^nY(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \ldots - sy^{(n-2)}(0) - y^{(n-1)}(0)
\end{aligned}$$

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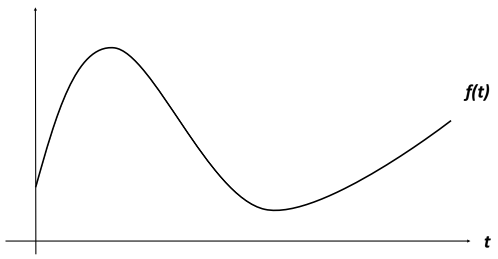

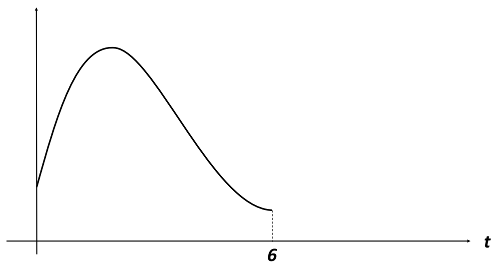

2. Find:

$$\mathscr{L}^{-1}\left\{\frac{e^{-3s}}{s^2+4}\right\}$$

3. Find:

$$\mathscr{L}\left\{(t-1)^3 e^t u_1(t)\right\}$$

Laplace transforms of some basic functions

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$\mathscr{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos(kt)$

$\mathscr{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} = u_a(t)$

$\mathscr{L}^{-1}\left\{\frac{1}{s^2-k^2}\right\} = \frac{1}{k}\sinh(kt)$

$\mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} = \cosh(kt)$

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$$\mathscr{L}^{-1}\{F(s-k)\} = \mathscr{L}^{-1}\left\{F(s)|_{s\to s-k}\right\} = e^{kt}\mathscr{L}^{-1}\{F(s)\} = e^{kt}f(t)$$

Translation Theorem II:

$$\mathscr{L}\left\{f(t-a)u_a(t)\right\} = e^{-as}F(s) = e^{-as}\mathscr{L}\{f(t)\}$$

$$\mathscr{L}^{-1}\{e^{-as}F(s)\} = f(t-a)u_a(t) = \mathscr{L}^{-1}\{F(s)\}\Big|_{t\to t-a}u_a(t)$$

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\end{aligned}$$