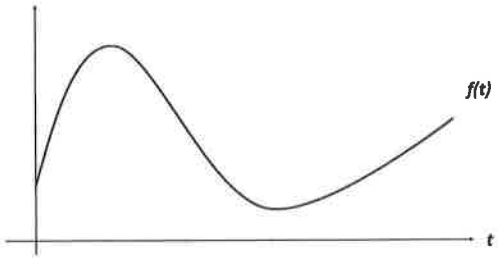
NAME: Solutions

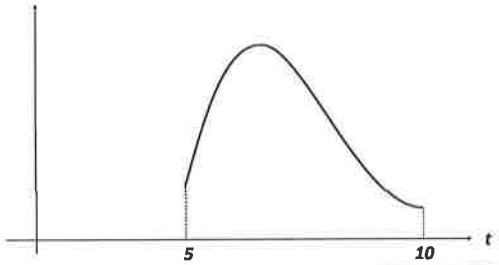
Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 9

1. In the picture to the left below there is the graph of a function f. To the right is the graph obtained by shifting the graph of f by 5 units to the right, then "turning off" the result at $t = 10$. Assuming the function to the right is 0 everywhere except on the interval $[5, 10]$, express it in terms of f and unit step functions.

(3 pts.)





$$f(t-5)\mathbb{1}_{[5,10]}(t) = \boxed{f(t-5)(u_5(t)-u_{10}(t))}$$

2. Find:

(3.5 pts.)

$$\mathscr{L}^{-1}\left\{\frac{se^{-4s}}{s^2+9}\right\} = \mathscr{L}^{-1}\left\{\frac{s}{s^2+9}\right\}\Big|_{t\to t-4} u_4(t)$$

$$= \cos(3t)\Big|_{t\to t-4} u_4(t) = \boxed{\cos(3t-12)\,u_4(t)}$$

3. Find:

$$\mathscr{L}\{e^{t-2}(u_2(t)-u_5(t))\} = \mathscr{L}\{e^{t-2}u_2(t)\} - \mathscr{L}\{e^{t-2}u_5(t)\}$$

$$= e^{-2s}\mathscr{L}\{e^t\} - \mathscr{L}\{e^{t-5+3}u_5(t)\}$$

(3.5 pts.)

$$= e^{-2s}\frac{1}{s-1} - e^{-5s}\mathscr{L}\{e^{t+3}\}$$

$$= \boxed{e^{-2s}\frac{1}{s-1} - e^{-5s}\frac{e^3}{s-1}}$$

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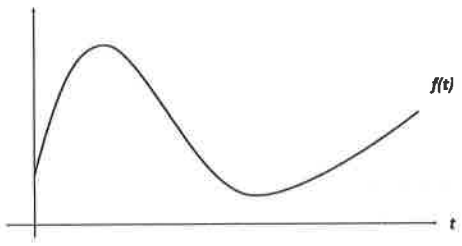
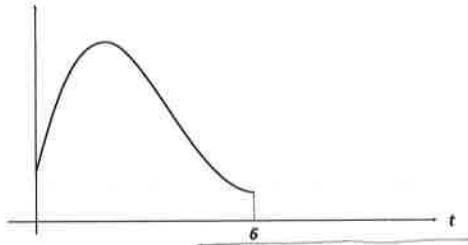
Quiz 9

1. In the picture to the left below there is the graph of a function f. To the right is the graph obtained by "turning off" the graph of f at $t = 6$. Assuming the function to the right is 0 everywhere except on the interval $[0, 6]$, express it in terms of f and unit step functions.

(3 pts.)





$$f(t)\mathbb{1}_{[0,6]}(t) = f(t)\left(u_0(t) - u_6(t)\right)$$
$$= f(t)\left(1 - u_6(t)\right)$$

2. Find:

(3.5 pts.)

$$\mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}\Big|_{t\to t-3}\, u_3(t)$$
$$= \frac{1}{2}\sin(2t)\Big|_{t\to t-3}\, u_3(t) = \frac{1}{2}\sin(2t-6)\,u_3(t)$$

3. Find:

$$\mathcal{L}\left\{(t-1)^3 e^t u_1(t)\right\} = \mathcal{L}\left\{(t-1)^3 e^{t-1} e\, u_1(t)\right\}$$
$$= e^{-s} e\, \mathcal{L}\left\{t^3 e^t\right\}$$

(3.5 pts.)

$$= e^{1-s}\, \mathcal{L}\left\{t^3\right\}\Big|_{s\to s-1}$$
$$= e^{1-s}\, \frac{3!}{s^4}\Big|_{s\to s-1}$$
$$= e^{1-s}\, \frac{6}{(s-1)^4}$$