NAME: ....

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 8

Find the inverse Laplace transforms below:

1. $\mathscr{L}^{-1}\left\{\frac{s+4}{s^2+4s+8}\right\}$

2. $\mathscr{L}^{-1}\left\{\frac{s+3}{(s-2)(s+1)}\right\}$

3. $\mathscr{L}^{-1}\left\{\frac{3s^2-s+6}{(s+1)(s^2+9)}\right\}$

Laplace transforms of some basic functions

$\mathscr{L}\{1\} = \frac{1}{s};\ \ s > 0$

$\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}};\ \ s > 0$

$\mathscr{L}\{e^{kt}\} = \frac{1}{s-k};\ \ s > k$

$\mathscr{L}\{\sin(kt)\} = \frac{k}{s^2+k^2};\ \ s > 0$

$\mathscr{L}\{\cos(kt)\} = \frac{s}{s^2+k^2};\ \ s > 0$

$\mathscr{L}\{\sinh(kt)\} = \frac{k}{s^2-k^2};\ \ s > |k|$

$\mathscr{L}\{\cosh(kt)\} = \frac{s}{s^2-k^2};\ \ s > |k|$

Inverse Laplace transforms of some basic functions

$\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$\mathscr{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{1}{(n-1)!}t^{n-1}$

$\mathscr{L}^{-1}\left\{\frac{1}{s-k}\right\} = e^{kt}$

$\mathscr{L}^{-1}\left\{\frac{1}{s^2+k^2}\right\} = \frac{1}{k}\sin(kt)$

$\mathscr{L}^{-1}\left\{\frac{s}{s^2+k^2}\right\} = \cos(kt)$

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$\mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\} = \cosh(kt)$

Properties of the Laplace and Inverse Laplace transform

Translation Theorem:

$$\mathscr{L}\left\{e^{kt}f(t)\right\} = F(s-k) = \mathscr{L}\{f(t)\}|_{s\to s-k}$$

$$\mathscr{L}^{-1}\{F(s-k)\} = \mathscr{L}^{-1}\left\{F(s)|_{s\to s-k}\right\} = e^{kt}f(t) = e^{kt}\mathscr{L}^{-1}\{F(s)\}$$

Derivatives of Laplace Transforms:

$$\mathscr{L}\left\{t^n f(t)\right\} = (-1)^n\frac{d^n}{ds^n}F(s)$$

$$\mathscr{L}^{-1}\left\{F^{(n)}(s)\right\} = (-1)^n t^n f(t)$$

Laplace Transforms of Derivatives:

$$\begin{aligned}
\mathscr{L}\{y'\} &= sY(s) - y(0)\\
\mathscr{L}\{y''\} &= s^2Y(s) - sy(0) - y'(0)\\
\mathscr{L}\{y'''\} &= s^3Y(s) - s^2y(0) - sy'(0) - y''(0)\\
&\ \vdots\\
\mathscr{L}\{y^{(n)}(t)\} &= s^nY(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \ldots - sy^{(n-2)}(0) - y^{(n-1)}(0)
\end{aligned}$$

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Find the inverse Laplace transforms below:

1. $\mathscr{L}^{-1}\left\{\dfrac{s+3}{s^2+2s+5}\right\}$

2. $\mathscr{L}^{-1}\left\{\dfrac{2s+3}{s(s+3)}\right\}$

3. $\mathscr{L}^{-1}\left\{\dfrac{8s^2-4s+12}{s(s^2+4)}\right\}$

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$\mathscr{L}\{e^{kt}\} = \frac{1}{s-k}; \ s > k$

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$\mathscr{L}\{\sinh(kt)\} = \frac{k}{s^2-k^2}; \ s > |k|$

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$\mathscr{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{1}{(n-1)!}t^{n-1}$

$\mathscr{L}^{-1}\left\{\frac{1}{s-k}\right\} = e^{kt}$

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$$\mathscr{L}^{-1}\{F(s-k)\} = \mathscr{L}^{-1}\left\{F(s)|_{s\to s-k}\right\} = e^{kt}f(t) = e^{kt}\mathscr{L}^{-1}\{F(s)\}$$

Derivatives of Laplace Transforms:

$$\mathscr{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n}F(s)$$

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&\vdots \\
\mathscr{L}\{y^{(n)}(t)\} &= s^nY(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \ldots - sy^{(n-2)}(0) - y^{(n-1)}(0)
\end{aligned}$$