NAME: Solutions

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 8

Find the inverse Laplace transforms below:

1. $\mathscr{L}^{-1}\left\{\dfrac{s+4}{s^2+4s+8}\right\}$ **2.** $\mathscr{L}^{-1}\left\{\dfrac{s+3}{(s-2)(s+1)}\right\}$ **3.** $\mathscr{L}^{-1}\left\{\dfrac{3s^2-s+6}{(s+1)(s^2+9)}\right\}$

① 3.5 pts.

$$\mathcal{L}^{-1}\left\{\frac{s+4}{s^2+4s+8}\right\} = \mathcal{L}^{-1}\left\{\frac{s+2+2}{(s+2)^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+4}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{(s+2)^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}\Big|_{s\to s+2} + 2\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}\Big|_{s\to s+2}$$

$$= e^{-2t}\cos(2t) + 2e^{-2t}\frac{1}{2}\sin(2t) = \boxed{e^{-2t}\left(\cos(2t)+\sin(2t)\right)}$$

(1 pt.) – completing the square

(1.5 pts.) – breaking up the numerator & applying the Translation Thm. correctly

(1 pt.) – final answer

② 3 pts.

$$\frac{s+3}{(s-2)(s+1)} = \frac{A}{s-2} + \frac{B}{s+1}; \quad \frac{s+3}{s+1}\Big|_{s=2} = A; \quad \frac{s+3}{s-2}\Big|_{s=-1} = B$$

$A = 5/3$ $B = -2/3$

$$\mathcal{L}^{-1}\left\{\frac{s+3}{(s-2)(s+1)}\right\} = \frac{5}{3}\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\} - \frac{2}{3}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = \boxed{\frac{5}{3}e^{2t} - \frac{2}{3}e^{-t}}$$

(2 pts.) – partial fraction decomposition

(1 pt.) – final answer.

③ 3.5 pts.

$$\frac{3s^2-s+6}{(s+1)(s^2+9)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+9}; \quad \frac{3s^2-s+6}{s^2+9}\Big|_{s=-1} = A \Rightarrow A=1$$

$$\Rightarrow \frac{Bs+C}{s^2+9} = \frac{3s^2-s+6}{(s+1)(s^2+9)} - \frac{1}{s+1} = \frac{3s^2-s+6-s^2-9}{(s+1)(s^2+9)} = \frac{2s^2-s-3}{(s+1)(s^2+9)} = \frac{(s+1)(2s-3)}{(s+1)(s^2+9)}$$

$$= \frac{2s-3}{s^2+9} \Rightarrow B=2 \quad C=-3$$

$$\mathcal{L}^{-1}\left\{\frac{3s^2-s+6}{(s+1)(s^2+9)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 2\mathcal{L}^{-1}\left\{\frac{s}{s^2+9}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\}$$

$$= \boxed{e^{-t} + 2\cos(3t) - \sin(3t)}$$

(2.5 pts.) – partial fraction decomposition

(1 pt.) – final answer.

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Find the inverse Laplace transforms below:

1. $\mathscr{L}^{-1}\left\{\dfrac{s+3}{s^2+2s+5}\right\}$ **2.** $\mathscr{L}^{-1}\left\{\dfrac{2s+3}{s(s+3)}\right\}$ **3.** $\mathscr{L}^{-1}\left\{\dfrac{8s^2-4s+12}{s(s^2+4)}\right\}$

(1) (3.5) pts.

$$\mathcal{L}^{-1}\left\{\frac{s+3}{s^2+2s+5}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1+2}{(s+1)^2+4}\right\} = \mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+4}\right\} + 2\mathcal{L}^{-1}\left\{\frac{1}{(s+1)^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}\Big|_{s\to s+1} + 2\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}\Big|_{s\to s+1}$$

$$= e^{-t}\cos(2t) + 2\cdot e^{-t}\tfrac{1}{2}\sin(2t) = \boxed{e^{-t}(\cos(2t)+\sin(2t))}.$$

(1pt.) – completing the square
(1.5 pts.) – breaking up the numerator & applying the Translation Thm. correctly
(1pt.) – final answer.

(2) (3) pts.

$$\frac{2s+3}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}; \quad \frac{2s+3}{s+3}\Big|_{s=0} = A; \quad \frac{2s+3}{s}\Big|_{s=-3} = B$$

$A=1$ $B=1$

$$\mathcal{L}^{-1}\left\{\frac{2s+3}{s(s+3)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} = 1 + e^{-3t}.$$

(2pts.) – partial fraction decomposition
(1pt.) – final answer.

(3) (3.5) pts.

$$\frac{8s^2-4s+12}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}; \quad \frac{8s^2-4s+12}{s^2+4}\Big|_{s=0} = A \Rightarrow A=3$$

$$\Rightarrow \frac{Bs+C}{s^2+4} = \frac{8s^2-4s+12}{s(s^2+4)} - \frac{3}{s} = \frac{8s^2-4s+\cancel{12}-3s^2-\cancel{12}}{s(s^2+4)} = \frac{s(5s-4)}{s(s^2+4)}$$

$$= \frac{5s-4}{s^2+4} \Rightarrow B=5 \quad C=-4$$

$$\mathcal{L}^{-1}\left\{\frac{8s^2-4s+12}{s(s^2+4)}\right\} = 3\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + 5\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} - 4\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}$$

$$= \boxed{3 + 5\cos(2t) - 2\sin(2t)}$$

(2.5pts.) – partial fraction decomposition
(1pt.) – final answer