NAME: Solutions

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 7

Laplace transforms of some basic functions:

$\mathscr{L}\{1\} = \frac{1}{s}$; $s > 0$.

$\mathscr{L}\{\sin(kt)\} = \frac{k}{s^2+k^2}$; $s > 0$.

$\mathscr{L}\{\sinh(kt)\} = \frac{k}{s^2-k^2}$; $s > |k|$.

$\mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}}$; $s > 0$.

$\mathscr{L}\{\cosh(kt)\} = \frac{s}{s^2-k^2}$; $s > |k|$.

$\mathscr{L}\{\cos(kt)\} = \frac{s}{s^2+k^2}$; $s > 0$.

$\mathscr{L}\{e^{kt}\} = \frac{1}{s-k}$; $s > k$.

Properties of the Laplace transform:

Translation Theorem: $\boxed{\mathscr{L}\left\{e^{kt}f(t)\right\} = F(s-k) = \mathscr{L}\{f(t)\}|_{s\to s-k}}$

Derivatives of Laplace Transforms: $\boxed{\mathscr{L}\left\{t^n f(t)\right\} = (-1)^n \frac{d^n}{ds^n}F(s)}$

Laplace Transforms of Derivatives: $\boxed{\mathscr{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)}$

Find the Laplace transforms below, with the frequency domain as well.

1. $\mathscr{L}\{te^{4t}\}$.

2. $\mathscr{L}\{e^{-3t}\cos(7t)\}$.

3. $\mathscr{L}\left\{\frac{e^{\sqrt{2}t} - e^{-\sqrt{2}t}}{2}\right\}$.

(4 pts.) 1. $\mathscr{L}\{te^{4t}\} = \mathscr{L}\{t\}|_{s\to s-4} = \frac{1}{s^2}\Big|_{s\to s-4} = \frac{1}{(s-4)^2}$; $(s>4)$

$(s>0) \to (s-4>0)$ (3 pts.) (1 pt.)

(4 pts.) 2. $\mathscr{L}\{e^{-3t}\cos(7t)\} = \mathscr{L}\{\cos(7t)\}|_{s\to s+3} = \frac{s}{s^2+49}\Big|_{s\to s+3} = \frac{s+3}{(s+3)^2+49}$; $(s>-3)$

$(s>0) \to (s+3>0)$ (3 pts.) (1 pt.)

(2 pts.) 3. $\mathscr{L}\{\sinh(\sqrt{2}t)\} = \frac{\sqrt{2}}{s^2-2}$; $s>\sqrt{2}$.

(1 pt.) (1 pt.)

or, the "hard" way:

$\frac{1}{2}\mathscr{L}\{e^{\sqrt{2}t}\} - \frac{1}{2}\mathscr{L}\{e^{-\sqrt{2}t}\} = \frac{1}{2}\frac{1}{s-\sqrt{2}} - \frac{1}{2}\frac{1}{s+\sqrt{2}} = \frac{1}{2}\frac{s+\sqrt{2}-s+\sqrt{2}}{s^2-2} = \frac{\sqrt{2}}{s^2-2}$

$(s>\sqrt{2}) \cap (s>-\sqrt{2})$ $(s>\sqrt{2})$

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Find the Laplace transforms below, with the frequency domain as well.

1. $\mathscr{L}\{te^{-8t}\}$. **2.** $\mathscr{L}\{e^{2t}\sin(3t)\}$. **3.** $\mathscr{L}\left\{\frac{e^{10t}+e^{-10t}}{2}\right\}$.

(4pts.) ① $\mathscr{L}\{te^{-8t}\} = \mathscr{L}\{t\}|_{s\to s+8} = \frac{1}{s^2}\Big|_{s\to s+8} = \frac{1}{(s+8)^2}$; $(s>-8)$

$(s>0)\to(s+8>0)$ (3 pts.) (1 pt.)

(4pts.) ② $\mathscr{L}\{e^{2t}\sin(3t)\} = \mathscr{L}\{\sin(3t)\}|_{s\to s-2} = \frac{3}{s^2+9}\Big|_{s\to s-2} = \frac{3}{(s-2)^2+9}$; $(s>2)$

$(s>0)\to(s-2>0)$ (3pts.) (1 pt.)

(2pts.) ③ $\mathscr{L}\{\cosh(10t)\} = \frac{s}{s^2-100}$; $s>10$

(1pt.) (1pt.)

or, the "hard" way:

$\frac{1}{2}\mathscr{L}\{e^{10t}\} + \frac{1}{2}\mathscr{L}\{e^{-10t}\} = \frac{1}{2}\frac{1}{s-10} + \frac{1}{2}\frac{1}{s+10} = \frac{1}{2}\frac{s+10+s-10}{s^2-100} = \frac{s}{s^2-100}$

$(s>10) \cap (s>-10)$ $(s>10)$