NAME: Solutions

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 6

[3] **1.** Find the general solution of the differential equation:

$$x^2y'' + 3xy' + 3y = 0;\ x > 0.$$

(1pt) Cauchy-Euler; $a=1;\ b=c=3$

(1pt) Char. Eqn.: $m^2+2m+3=0 \Rightarrow m = \dfrac{-2 \pm \sqrt{-8}}{2} = -1 \pm i\sqrt{2}$

(1pt) $$\boxed{y = \frac{1}{x}\left(c_1 \cos(\sqrt{2}\ln x) + c_2 \sin(\sqrt{2}\ln x)\right)}$$

[7] **2.** Find the general solution of the differential equation:

$$y'' + y = \sec x;\ x \in (-\pi/2, \pi/2).$$

(2pts.)

Complementary Solution: $m^2+1=0;\ m=\pm i$

$$\boxed{y_c = c_1 \sin x + c_2 \cos x}$$

Particular Solution: (Variation of parameters):

(1pt) $$W = \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix} = -1$$

(1pt) $$W_1 = \begin{vmatrix} 0 & \cos x \\ \sec x & -\sin x \end{vmatrix} = -1 \Rightarrow u_1' = \frac{-1}{-1} = 1 \Rightarrow \boxed{u_1 = x}$$ (0.5pt.)

(1pt) $$W_2 = \begin{vmatrix} \sin x & 0 \\ \cos x & \sec x \end{vmatrix} = \tan x \Rightarrow u_2' = -\tan x \Rightarrow \boxed{u_2 = \ln(\cos x)}$$ (1pt.)

$\cos x > 0$ on $(-\pi/2, \pi/2)$

$$\boxed{y_p = x\sin x + \cos x \ln(\cos x)}$$ (0.5pt.)

$$\Rightarrow \boxed{y = c_1 \sin x + c_2 \cos x + x\sin x + \cos x \cdot \ln(\cos x)}$$

NAME: Solutions

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Quiz 6

[3] **1.** Find the general solution of the differential equation:

$$x^2y'' - xy' + y = 0;\ x > 0.$$

(1 pt.) Cauchy-Euler: $a=1, b=-1, c=1$

(1 pt.) Char. Eq.: $m^2-2m+1=0$; $(m-1)^2=0$

(1 pt.) $$\boxed{y = c_1 x + c_2 x \ln x}$$

[7] **2.** Find the general solution of the differential equation:

$$y'' - 2y' + y = \frac{e^x}{x};\ x > 0.$$

(2 pts.) Complementary Solution: $m^2-2m+1=0$; $(m-1)^2=0$

$$\boxed{y_c = c_1 e^x + c_2 x e^x}$$

Particular Solution: (Variation of parameters)

(1 pt.) $$W = \begin{vmatrix} e^x & xe^x \\ e^x & (x+1)e^x \end{vmatrix} = e^{2x}$$

(1 pt.) $$W_1 = \begin{vmatrix} 0 & xe^x \\ \frac{1}{x}e^x & (x+1)e^x \end{vmatrix} = -e^{2x} \Rightarrow u_1' = -1 \Rightarrow \boxed{u_1 = -x}$$ (0.5 pt.)

(1 pt.) $$W_2 = \begin{vmatrix} e^x & 0 \\ e^x & \frac{1}{x}e^x \end{vmatrix} = \frac{1}{x}e^{2x} \Rightarrow u_2' = \frac{1}{x} \Rightarrow \boxed{u_2 = \ln x}$$ (0.5 pt.)

$$\boxed{y_p = -xe^x + xe^x \ln x}$$ (0.5 pt.)

$$\Rightarrow \boxed{y = c_1 e^x + c_2 x e^x + xe^x \ln x}$$ (0.5 pt.) – for removing redundant part. Sol.

absorbs $-xe^x$