NAME: Solutions

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 5

Find the general solution of the differential equation:

$$y'' - 4y' + 4y = 2 + e^{2x}.$$

(3 pts.) Complementary Sol. : $m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m_1 = m_2 = 2$

$$\boxed{y_c = c_1 e^{2x} + c_2 x e^{2x}}$$

Particular Sol. : $\boxed{y_p = A + Bx^2 e^{2x}}$ – (3 pts.) (1 pt.) for the "A" (2 pts.) for the "Bx^2e^{2x}"

$y_p' = 2Bxe^{2x} + 2Bx^2e^{2x}$

$y_p'' = 2Be^{2x} + 4Bxe^{2x} + 4Bxe^{2x} + 4Bx^2e^{2x}$

$y_p'' = 2Be^{2x} + 8Bxe^{2x} + 4Bx^2e^{2x}$

$y_p'' - 4y_p' + 4y_p = 2Be^{2x} + (8B - 8B)xe^{2x}$

$+ (4B - 8B + 4B)x^2e^{2x} + 4A$

$= 2Be^{2x} + 4A$

$= e^{2x} + 2$

$\begin{cases} 4A = 2 \\ 2B = 1 \end{cases}$

$\begin{cases} A = 1/2 \\ B = 1/2 \end{cases}$

(4 pts.) $\boxed{y_p = \frac{1}{2} + \frac{1}{2}x^2e^{2x}}$

General Sol. : $\boxed{y = c_1e^{2x} + c_2xe^{2x} + \frac{1}{2} + \frac{1}{2}x^2e^{2x}}$

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Find the general solution of the differential equation:

$$y'' - y' = xe^x.$$

(3pts.) Complementary Sol. : $m^2 - m = 0$, $m(m-1) = 0$, $m = 0, 1$

$$y_c = c_1 + c_2 e^x$$

Particular Sol. : $y_p = (Ax^2 + Bx)e^x$ – (3pts) (1 pt.) for multiplying by x; (2pts.) for "$(Ax+B)e^x$"

$$y_p' = (2Ax + B + Ax^2 + Bx)e^x = (Ax^2 + (2A+B)x + B)e^x$$

$$y_p'' = (2Ax + 2A + B + Ax^2 + (2A+B)x + B)e^x$$
$$= (Ax^2 + (4A+B)x + (2A+2B))e^x$$

$$y_p'' - y_p' = (2Ax + (2A+B))e^x = xe^x \Rightarrow \begin{cases} 2A = 1 \\ 2A + B = 0 \end{cases}$$

$A = 1/2$, $B = -1$

(4pts.)

$$y_p = \left(\frac{1}{2}x^2 - x\right)e^x$$

General Sol. :

$$y = c_1 + c_2 e^x + \left(\frac{1}{2}x^2 - x\right)e^x$$