Quiz 4 - Solutions (F)

① $\frac{dy}{dx} = \frac{x}{y} + \frac{y}{x}$

Homogeneous (degree 0) (1/2 pt. - recognize type/intent to solve as homogeneous)

Substitution: $\boxed{u = \frac{y}{x}}$ (1pt.) $\Rightarrow \boxed{\frac{dy}{dx} = u + x\frac{du}{dx}}$ (1/2 pt.)

$\Rightarrow \cancel{u} + x\frac{du}{dx} = \frac{1}{u} + \cancel{u}$ (1/2 pt.)

$\Rightarrow x\frac{du}{dx} = \frac{1}{u}$ (1/2 pt.)

$\Rightarrow u\,du = \frac{1}{x}dx$ (1/2 pt.)

$\Rightarrow \frac{u^2}{2} + c = \ln|x|$ (1 pt.)

$\Rightarrow x = ce^{u^2/2}$

$\Rightarrow \boxed{x = ce^{y^2/2x^2}}$ (1/2 pt.)

② $e^{x+y}dx - dy = 0$

Separable: (1/2 pt. - recognizing type)

$e^x e^y - \frac{dy}{dx} = 0$

$e^x e^y = \frac{dy}{dx}$

$e^x dx = e^{-y}dy$ 2 pts. - separating

$e^x + c = -e^{-y}$ 2 pts. - integrals

$e^x + e^{-y} = c$ 1/2 pt. - final answer

Quiz 4 - Solutions (L)

① $x\frac{dy}{dx} + (2x+1)y = e^{-2x}$; $x>0$

Linear (½ pt.)

Standard Form : $\frac{dy}{dx} + \left(2+\frac{1}{x}\right)y = \frac{1}{x}e^{-2x}$ (1 pt.)

Integrating Factor : $\int p(x)dx = \int\left(2+\frac{1}{x}\right)dx = 2x + \ln x$ (2 pts.)

$\mu(x) = e^{2x+\ln x} = e^{2x}e^{\ln x} = xe^{2x}$

Multiply by $\mu(x)$: $\frac{d}{dx}\left(xe^{2x}y\right) = 1$

$xe^{2x}y = x + c$ (1pt) $\boxed{y = e^{-2x}\left(1+\frac{c}{x}\right)}$ (½ pt.)

② $(4y+yx^2)dy - (2x+xy^2)dx = 0$

$N_x = 2xy$ $\qquad M_y = -2xy$ $\qquad$ (not exact)

$y(4+x^2)dy = x(2+y^2)dx$ Separable (1pt) - recognizing type

$\frac{y}{2+y^2}dy = \frac{x}{4+x^2}dx$ (1pt) - separating

$\frac{1}{2}\ln(2+y^2) = \frac{1}{2}\ln(4+x^2) + C$ (2pts.) - integrate

$\ln(2+y^2) = \ln(4+x^2) + C$

$2+y^2 = c(4+x^2)$

$\boxed{y^2 = c(4+x^2) - 2}$ (1pt.) - final answer