NAME: ....Solutions.....

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 2

1. [5 points] Determine whether or not the equation below is exact. If so, solve it:

$$(2xy - \sin x)\,dx + (x^2 + \cos y)\,dy = 0.$$

2. [5 points] Solve the equation:

$$\frac{dy}{dx} + y = \frac{1 - e^{-2x}}{e^x + e^{-x}}$$

(1) $\frac{\partial M}{\partial y} = 2x$ (1 pt. - testing for exactness)

$\frac{\partial N}{\partial x} = 2x$

=> The equation is exact.

Find potential: (1/2 pt. - intent to find potential)

$\frac{\partial f}{\partial x} = 2xy - \sin x$

$\Rightarrow f(x,y) = x^2y + \cos x + g(y)$

$\Rightarrow \frac{\partial f}{\partial y} = x^2 + g'(y)$

$= x^2 + \cos y$

$\Rightarrow g'(y) = \cos y$

$\Rightarrow g(y) = \sin y$

$f(x,y) = x^2y + \cos x + \sin y$ (3 pts. - finding a potential)

Solutions:

$\boxed{x^2y + \cos x + \sin y = c}$ (1/2 pt. - final answer)

(2) Linear 1st order ODE (1/2 pt. - intent to find int. factor)
=> use integrating factors

$p(x) = 1$ (1/2 pt. - identify p correctly)

$\int p(x)\,dx = x$ (1/2 pt.)

$\mu(x) = e^{\int p(x)dx}$ (1/2 pt.)

$\mu(x) = e^x$ (1/2 pt.)

Multiply by $\mu(x)$:

$\frac{d}{dx}(ye^x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ (1/2 pt.)

$ye^x = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} = \ln(e^x + e^{-x}) + c$ (1 1/2 pts.)

$\boxed{y = e^{-x}\ln(e^x + e^{-x}) + ce^{-x}}$ (1/2 pt. - final answer)

NAME: Solutions

Georgia Tech, Fall 2015
Math 2552 (Sections L1 – L4)

Quiz 2

1. [5 points] Determine whether or not the equation below is exact. If so, solve it:

$$(\cos x - \sin x + y^2)\,dx + 2xy\,dy = 0.$$

2. [5 points] Solve the equation:

$$(1+e^{2x})\frac{dy}{dx} + e^{2x}y = 0.$$

(1) $\frac{\partial M}{\partial y} = 2y$

$\frac{\partial N}{\partial x} = 2y$

(1 pt. - testing for exactness)

$\Rightarrow$ The equation is exact.

Find a potential function: (1/2 pt. - intent to find potential)

$\frac{\partial f}{\partial y} = 2xy \Rightarrow f(x,y) = xy^2 + g(x)$

$\Rightarrow \frac{\partial f}{\partial x} = y^2 + g'(x)$

$= y^2 + \cos x - \sin x$

$\Rightarrow g'(x) = \cos x - \sin x$

$\Rightarrow g(x) = \sin x + \cos x$

$f(x,y) = xy^2 + \sin x + \cos x$ (3 pts. - finding the potential)

Solutions:

$$\boxed{xy^2 + \sin x + \cos x = c}$$ (1/2 pt. - final answer)

(2) Standard form: (1/2 pt. - put in standard form)

$$\frac{dy}{dx} + \frac{e^{2x}}{1+e^{2x}}y = 0$$ (1/2 pt. - intent to find int. factor)

Linear 1st order $\Rightarrow$ use integrating factors

$p(x) = \frac{e^{2x}}{1+e^{2x}}$ (1/2 pt. - identify p correctly)

$\int p(x)\,dx = \int \frac{e^{2x}}{1+e^{2x}}\,dx = \frac{1}{2}\ln(1+e^{2x})$ (1 pt.)

$\mu(x) = e^{\int p(x)dx}$ (1/2 pt.)

$\mu(x) = e^{\frac{1}{2}\ln(1+e^{2x})} = \sqrt{1+e^{2x}}$ (1/2 pt.)

Multiply eqn. by $\mu(x)$:

$\Rightarrow \frac{d}{dx}\left(\sqrt{1+e^{2x}}\cdot y\right) = 0$ (1/2 pt.)

$\Rightarrow \sqrt{1+e^{2x}}\cdot y = c$ (1/2 pt.)

$$\Rightarrow \boxed{y = \frac{c}{\sqrt{1+e^{2x}}}}$$ (1/2 pt. - final answer)