Quiz 13

~ Solutions ~

(6 pts.)

1 $x \sin y \frac{dy}{dx} + \cos y = -x^2 e^x$

$u = \cos y \Rightarrow \frac{du}{dx} = -\sin y \frac{dy}{dx}$

$\Rightarrow -x \frac{du}{dx} + u = -x^2 e^x$ (Linear)

$\Rightarrow \frac{du}{dx} - \frac{1}{x} u = x e^x$

$\mu(x) = \frac{1}{x} \Rightarrow \frac{d}{dx}\left(\frac{1}{x} u\right) = e^x \Rightarrow \frac{1}{x} u = e^x + C$

$\Rightarrow \boxed{\cos y = x(e^x + C)}$

(4 pts.)

2 $y^2 y'' = y'$

$u = y' \Rightarrow y'' = \frac{du}{dx} = \frac{du}{dy} \frac{dy}{dx} = \frac{du}{dy} y' \Rightarrow \boxed{y'' = \frac{du}{dy} u}$

$\Rightarrow \boxed{y^2 \frac{du}{dy} u = u}$ If $u \equiv 0$, then $y' \equiv 0$, so $\boxed{y = C}$ (all constant functions are solutions).

If $u \not\equiv 0$: $y^2 \frac{du}{dy} = 1 \Rightarrow du = \frac{1}{y^2} dy \Rightarrow u = -\frac{1}{y} + c \Rightarrow \boxed{y' = -\frac{1}{y} + c}$ Autonomous

$\Rightarrow \frac{dy}{dx} = \frac{cy - 1}{y} \Rightarrow dx = \frac{y}{cy - 1} dy$

Case 1: $c = 0 \Rightarrow dx = -y\,dy \Rightarrow \boxed{x = -\frac{y^2}{2} + c_1}$

Case 2: $c \neq 0 \Rightarrow dx = \frac{1}{c} \frac{cy - 1 + 1}{cy - 1} dy = \frac{1}{c}\left(1 + \frac{1}{cy - 1}\right)$

$\Rightarrow \boxed{x = \frac{1}{c}\left(y + \frac{1}{c} \ln|cy - 1|\right) + c_2}$