NAME: ..Solutions.......................... Georgia Tech, Fall 2015
Math 2552 (Sections L1 – L4)

Quiz 12

(5pts.) **1.** Find the general solution of the linear system below, and express it in the form $\mathbf{x} = c_1\mathbf{x}_1 + c_2\mathbf{x}_2$, where $\mathbf{x}_1$ and $\mathbf{x}_2$ are two *real* solutions:

$$\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 5 & -5 \end{pmatrix}\mathbf{x}.$$

$$\begin{vmatrix} 1-\lambda & -2 \\ 5 & -5-\lambda \end{vmatrix} = (\lambda+5)(\lambda-1)+10 = \lambda^2+4\lambda+5 \qquad \lambda = \frac{-4\pm 2i}{2}$$

$$\Delta = 16-20 = -4 \qquad \boxed{\lambda = -2\pm i} \quad \text{(1.5 pts.)}$$

$$\lambda=-2-i: \left(\begin{array}{cc|c} 3+i & -2 & 0 \\ 5 & -3+i & 0 \end{array}\right) \Rightarrow v_1 = \frac{2}{3+i}v_2 \Rightarrow \boxed{\vec{v}_1 = \begin{pmatrix} 2 \\ 3+i \end{pmatrix}} \quad \text{(1.5pts.)}$$

$$\vec{x} = e^{-2t}(\cos t - i\sin t)\left(\begin{pmatrix} 2 \\ 3 \end{pmatrix} + i\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)$$

$$\vec{x}_1 = \mathrm{Re}(\vec{x}) = e^{-2t}\left(\begin{pmatrix} 2\cos t \\ 3\cos t \end{pmatrix} + \begin{pmatrix} 0 \\ \sin t \end{pmatrix}\right) = \boxed{e^{-2t}\begin{pmatrix} 2\cos t \\ 3\cos t+\sin t \end{pmatrix}} \quad \text{(1pt.)}$$

$$\vec{x}_2 = \mathrm{Im}(\vec{x}) = e^{-2t}\left(-\begin{pmatrix} 2\sin t \\ 3\sin t \end{pmatrix} + \begin{pmatrix} 0 \\ \cos t \end{pmatrix}\right) = \boxed{e^{-2t}\begin{pmatrix} -2\sin t \\ -3\sin t+\cos t \end{pmatrix}} \quad \text{(1pt.)}$$

(5pts.) **2.** Find the general solution of the linear system below:

$$\mathbf{x}' = \begin{pmatrix} 0 & -2 \\ 2 & 4 \end{pmatrix}\mathbf{x}.$$

$$\begin{vmatrix} -\lambda & -2 \\ 2 & 4-\lambda \end{vmatrix} = \lambda(\lambda-4)+4 = \lambda^2-4\lambda+4 = (\lambda-2)^2$$

(1pt.)

$$\lambda=2: \left(\begin{array}{cc|c} -2 & -2 & 0 \\ 2 & 2 & 0 \end{array}\right) \Rightarrow \vec{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{(1pt.)}$$

$$\begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow w_1+w_2 = -1/2 \Rightarrow \vec{w} = \begin{pmatrix} -1/2 \\ 0 \end{pmatrix} \quad \text{(2pts.)}$$

$$\boxed{\vec{x} = c_1 e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2\left(e^{2t}t\begin{pmatrix} 1 \\ -1 \end{pmatrix} + e^{2t}\begin{pmatrix} -1/2 \\ 0 \end{pmatrix}\right)} \quad \text{(1pt.)}$$

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$$\mathbf{x}' = \begin{pmatrix} 2 & -1 \\ 2 & 4 \end{pmatrix}\mathbf{x}.$$

$$\begin{vmatrix} 2-\lambda & -1 \\ 2 & 4-\lambda \end{vmatrix} = (\lambda-4)(\lambda-2)+2 = \lambda^2-6\lambda+10 \qquad \lambda = \frac{6\pm 2i}{2}$$

$$\Delta = 36-40 = -4 \qquad \boxed{\lambda = 3\pm i}$$

$$\lambda = 3+i: \left(\begin{array}{cc|c} -1-i & -1 & 0 \\ 2 & 1-i & 0 \end{array}\right) \Rightarrow (1+i)v_1 = -v_2 \Rightarrow v_1 = -\frac{1}{1+i}v_2 \qquad \boxed{\vec{v}_1 = \begin{pmatrix} -1 \\ 1+i \end{pmatrix}}$$

1.5pts. 1.5pts.

$$\vec{v} = e^{3t}\left(\cos(t)+i\sin(t)\right)\left[\begin{pmatrix} -1 \\ 1 \end{pmatrix} + i\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \qquad \vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2$$

$$\vec{x}_1 = e^{3t}\left[\begin{pmatrix} -\cos t \\ \cos t \end{pmatrix} - \begin{pmatrix} 0 \\ \sin t \end{pmatrix}\right] = \boxed{e^{3t}\begin{pmatrix} -\cos t \\ \cos t - \sin t \end{pmatrix}} \qquad (\mathrm{Re}(\vec{v})) \quad \text{1pt.}$$

$$\vec{x}_2 = e^{3t}\left[\begin{pmatrix} 0 \\ \cos t \end{pmatrix} + \begin{pmatrix} -\sin t \\ \sin t \end{pmatrix}\right] = \boxed{e^{3t}\begin{pmatrix} -\sin t \\ \cos t + \sin t \end{pmatrix}} \qquad (\mathrm{Im}(\vec{v})) \quad \text{1pt.}$$

(5pts.) **2.** Find the general solution of the linear system below:

$$\mathbf{x}' = \begin{pmatrix} -3 & -2 \\ 2 & 1 \end{pmatrix}\mathbf{x}.$$

$$\begin{vmatrix} -3-\lambda & -2 \\ 2 & 1-\lambda \end{vmatrix} = (\lambda-1)(\lambda+3)+4 = \lambda^2+2\lambda+1 = (\lambda+1)^2$$

1pt.

$$\lambda = -1: \left(\begin{array}{cc|c} -2 & -2 & 0 \\ 2 & 2 & 0 \end{array}\right) \Rightarrow v_1 = -v_2 \Rightarrow \boxed{\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}} \quad \text{1pt.}$$

$$\begin{pmatrix} -2 & -2 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \Rightarrow w_1 + w_2 = -\tfrac{1}{2} \Rightarrow \boxed{\vec{w} = \begin{pmatrix} -1/2 \\ 0 \end{pmatrix}} \quad \text{2pts.}$$

$$\boxed{\vec{x} = c_1 e^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2\left(te^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + e^{-t}\begin{pmatrix} -1/2 \\ 0 \end{pmatrix}\right)} \quad \text{1pt.}$$