NAME: ...Solutions...........................

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 11

Find the general solution of the linear system below:

$$\mathbf{x}' = \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 0 \\ 2 & 2 & 2 \end{pmatrix} \mathbf{x}.$$

Eigenvalues:

$$\begin{vmatrix} 2-\lambda & 1 & 2 \\ 0 & 1-\lambda & 0 \\ 2 & 2 & 2-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 2-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix} = (1-\lambda)\left((\lambda-2)^2-4\right)$$
$$= (1-\lambda)(\lambda-4)(\lambda)$$

$$\boxed{\lambda_1=1;\ \lambda_2=4;\ \lambda_3=0}$$ 1.5 pts. each

Eigenvectors: 1.5 pts. each

$\lambda_1=1$ $$[A-I|\vec{0}] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{array}{l} v_1+v_2=0 \\ v_3=0 \end{array}$$

$$\boxed{\vec{v_1} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}$$

$\lambda_2=4$ $$[A-4I|\vec{0}] = \left[\begin{array}{ccc|c} -2 & 1 & 2 & 0 \\ 0 & -3 & 0 & 0 \\ 2 & 2 & -2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -1/2 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3/2 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$v_2=0$; $v_1=v_3$

$$\boxed{\vec{v_2} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}$$

$\lambda_3=0$ $$[A|\vec{0}] = \left[\begin{array}{ccc|c} 2 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 \end{array}\right]$$ $v_2=0$; $v_1+v_3=0$ $$\boxed{\vec{v_3} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}$$

Solution: 1pt.

$$\boxed{\vec{x} = c_1e^{t}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + c_2e^{4t}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_3\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}$$

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Quiz 11

Find the general solution of the linear system below:

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 & 4 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} \mathbf{x}.$$

Eigenvalues: $\begin{vmatrix} 1-\lambda & 1 & 4 \\ 0 & 2-\lambda & 0 \\ 1 & 1 & 1-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} = (2-\lambda)\left((\lambda-1)^2-4\right)$

$= (2-\lambda)(\lambda-3)(\lambda+1)$

$\boxed{\lambda_1=2;\ \lambda_2=3;\ \lambda_3=-1}$ 1.5 pts. each

Eigenvectors: 1.5 pts. each

$\lambda_1=2$: $[A-2I|\vec{0}] = \left[\begin{array}{ccc|c} -1 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & -4 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & -4 & 0 \\ 0 & +2 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & -4 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & 0 & -5/2 & 0 \\ 0 & 1 & 3/2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\quad v_1 = 5/2\, v_3$, $\quad v_2 = -3/2\, v_3$ $\quad \boxed{\vec{v}_1 = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}}$

$\lambda_2=3$: $[A-3I|\vec{0}] = \left[\begin{array}{ccc|c} -2 & 1 & 4 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1/2 & -2 & 0 \\ 1 & 1 & -2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$ $\Rightarrow v_1 = 2v_3$, $\Rightarrow v_2 = 0$ $\quad \boxed{\vec{v}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}}$

$\lambda_3=-1$: $[A+I|\vec{0}] = \left[\begin{array}{ccc|c} 2 & 1 & 4 & 0 \\ 0 & 3 & 0 & 0 \\ 1 & 1 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1/2 & 2 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$ $\Rightarrow v_1 = -2v_3$, $\Rightarrow v_2 = 0$ $\quad \boxed{\vec{v}_3 = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}}$

Solution: 1 pt.

$$\boxed{\vec{x} = c_1 e^{2t}\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} + c_2 e^{3t}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + c_3 e^{-t}\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}}$$