

NAME:

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 10

1. Solve the equation for $y(t)$:

$$y(t) = t + \int_0^t y(\tau) \sin(t - \tau) d\tau.$$

2. Solve the equation for $y(t)$:

$$y'' + 2y' = \delta(t - 1); y(0) = 0; y'(0) = 1.$$

Laplace transforms of some basic functions

$$\begin{aligned} \mathcal{L}\{1\} &= \frac{1}{s}; \quad s > 0 & \mathcal{L}\{\sin(kt)\} &= \frac{k}{s^2 + k^2}; \quad s > 0 & \mathcal{L}\{\cosh(kt)\} &= \frac{s}{s^2 - k^2}; \quad s > |k| \\ \mathcal{L}\{t^n\} &= \frac{n!}{s^{n+1}}; \quad s > 0 & \mathcal{L}\{\cos(kt)\} &= \frac{s}{s^2 + k^2}; \quad s > 0 & \mathcal{L}\{u_a(t)\} &= \frac{e^{-as}}{s}; \quad s > 0 \\ \mathcal{L}\{e^{kt}\} &= \frac{1}{s - k}; \quad s > k & \mathcal{L}\{\sinh(kt)\} &= \frac{k}{s^2 - k^2}; \quad s > |k| & \mathcal{L}\{\delta(t - t_0)\} &= e^{-st_0} \\ & & & & \mathcal{L}\{\delta(t)\} &= 1 \end{aligned}$$

Inverse Laplace transforms of some basic functions

$$\begin{aligned} \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} &= 1 & \mathcal{L}^{-1}\left\{\frac{1}{s^2 + k^2}\right\} &= \frac{1}{k} \sin(kt) & \mathcal{L}^{-1}\left\{\frac{s}{s^2 - k^2}\right\} &= \cosh(kt) \\ \mathcal{L}^{-1}\left\{\frac{1}{s^n}\right\} &= \frac{1}{(n-1)!} t^{n-1} & \mathcal{L}^{-1}\left\{\frac{s}{s^2 + k^2}\right\} &= \cos(kt) & \mathcal{L}^{-1}\left\{\frac{e^{-as}}{s}\right\} &= u_a(t) \\ \mathcal{L}^{-1}\left\{\frac{1}{s - k}\right\} &= e^{kt} & \mathcal{L}^{-1}\left\{\frac{1}{s^2 - k^2}\right\} &= \frac{1}{k} \sinh(kt) & \mathcal{L}^{-1}\{e^{-st_0}\} &= \delta(t - t_0) \\ & & & & \mathcal{L}^{-1}\{1\} &= \delta(t) \end{aligned}$$

Properties of the Laplace and Inverse Laplace transform

Translation Theorem I:

$$\begin{aligned} \mathcal{L}\{e^{kt} f(t)\} &= F(s - k) = \mathcal{L}\{f(t)\}|_{s \rightarrow s - k} \\ \mathcal{L}^{-1}\{F(s - k)\} &= \mathcal{L}^{-1}\{F(s)|_{s \rightarrow s - k}\} = e^{kt} \mathcal{L}^{-1}\{F(s)\} = e^{kt} f(t) \end{aligned}$$

Translation Theorem II:

$$\begin{aligned} \mathcal{L}\{f(t - a)u_a(t)\} &= e^{-as} F(s) = e^{-as} \mathcal{L}\{f(t)\} \\ \mathcal{L}^{-1}\{e^{-as} F(s)\} &= f(t - a)u_a(t) = \mathcal{L}^{-1}\{F(s)\}|_{t \rightarrow t - a} u_a(t) \end{aligned}$$

Derivatives of Laplace Transforms:

$$\begin{aligned} \mathcal{L}\{t^n f(t)\} &= (-1)^n \frac{d^n}{ds^n} F(s) \\ \mathcal{L}^{-1}\{F^{(n)}(s)\} &= (-1)^n t^n f(t) \end{aligned}$$

Laplace Transform of Periodic Functions:

If $f(t)$ is piecewise continuous on $[0, \infty)$, of exponential order, and periodic with period T :

$$\mathcal{L}\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$

Laplace Transforms of Derivatives:

$$\begin{aligned} \mathcal{L}\{y'\} &= sY(s) - y(0) \\ \mathcal{L}\{y''\} &= s^2 Y(s) - sy(0) - y'(0) \\ &\vdots \\ \mathcal{L}\{y^{(n)}(t)\} &= s^n Y(s) - s^{n-1} y(0) - s^{n-2} y'(0) - \dots - sy^{(n-2)}(0) - y^{(n-1)}(0) \end{aligned}$$

Convolution Theorem:

$$\begin{aligned} \mathcal{L}\{f(t) \star g(t)\} &= F(s)G(s) = \mathcal{L}\{f(t)\} \mathcal{L}\{g(t)\} \\ \mathcal{L}^{-1}\{F(s)G(s)\} &= f(t) \star g(t) \end{aligned}$$

Dirac Delta Function:

$$\begin{aligned} \int_0^\infty f(t) \delta(t - t_0) dt &= f(t_0) \\ (f \star \delta)(t) &= f(t) \end{aligned}$$

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1. Solve the equation for $y(t)$:

$$y(t) = 2 - \int_0^t y(\tau)e^{t-\tau} d\tau.$$

2. Solve the equation for $y(t)$:

$$y'' + 5y' + 6y = \delta(t - 1); y(0) = 0; y'(0) = 0.$$

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