NAME: ...Solutions.....

Georgia Tech, Fall 2015
Math 2552 (Sections L1 – L4)

Quiz 10

(5pts.) **1.** Solve the equation for $y(t)$:

$$y(t) = 2 - \int_0^t y(\tau) e^{t-\tau}\, d\tau.$$

$= 2 - y * e^t$ (+1)

(+1)
$$Y(s) = \mathcal{L}\{2\} - \mathcal{L}\{y * e^t\}$$
$$= \frac{2}{s} - Y(s)\,\frac{1}{s-1} \quad \Rightarrow \quad Y(s)\left(1 + \frac{1}{s-1}\right) = \frac{2}{s}$$
$$\Rightarrow \quad \frac{s}{s-1}\,Y(s) = \frac{2}{s} \quad \Rightarrow \quad \boxed{Y(s) = \frac{2(s-1)}{s^2}}$$ (+1.5)

$$Y(s) = \frac{2}{s} - \frac{2}{s^2} \Rightarrow y(t) = \mathcal{L}^{-1}\left\{\frac{2}{s} - \frac{2}{s^2}\right\} = 2 - 2t$$

$\boxed{y(t) = 2 - 2t}$ (+1.5)

(5pts.) **2.** Solve the equation for $y(t)$: $y'' + 5y' + 6y = \delta(t-1)\,;\; y(0) = 0,\, y'(0) = 0.$

(+1)
$$s^2 Y(s) + 5sY(s) + 6Y(s) = e^{-s}$$
$$(s^2 + 5s + 6)\,Y(s) = e^{-s} \quad \Rightarrow \quad Y(s) = \frac{e^{-s}}{s^2 + 5s + 6}$$ (+1)

$$\frac{1}{s^2+5s+6} = \frac{1}{(s+2)(s+3)}$$
$$= \frac{(s+3)-(s+2)}{(s+2)(s+3)}$$
$$= \frac{1}{s+2} - \frac{1}{s+3}$$ (+1)

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2+5s+6}\right\} = e^{-2t} - e^{-3t}$$ (+1)

$$y(t) = \mathcal{L}^{-1}\left\{\frac{e^{-s}}{s^2+5s+6}\right\}$$
$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2+5s+6}\right\}\Big|_{t \to t-1}\, u_1(t)$$
$$= \boxed{\left(e^{-2t+2} - e^{-3t+3}\right) u_1(t).}$$ (+1)

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Math 2552 (Sections F1 – F4)

Quiz 10

(5pts) **1.** Solve the equation for $y(t)$:

$$y(t) = t + \int_0^t y(\tau)\sin(t-\tau)\,d\tau.$$

$$= t + y * \sin(t)$$ (+1)

(+1) $Y(s) = \mathcal{L}\{t\} + \mathcal{L}\{y * \sin(t)\}$

$$= \frac{1}{s^2} + Y(s)\frac{1}{s^2+1} \Rightarrow \left(1 - \frac{1}{s^2+1}\right)Y(s) = \frac{1}{s^2}$$

$$\Rightarrow \frac{s^2}{s^2+1}Y(s) = \frac{1}{s^2} \Rightarrow \boxed{Y(s) = \frac{s^2+1}{s^4}}$$ (+1.5)

$$Y(s) = \frac{1}{s^2} + \frac{1}{s^4} \Rightarrow y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2} + \frac{1}{s^4}\right\} = t + \frac{1}{3!}t^3$$

$$\boxed{y(t) = t + \frac{1}{6}t^3}$$ (+1.5)

(5pts) **2.** Solve the equation for $y(t)$:

$$y'' + 2y' = \delta(t-1);\ y(0) = 0;\ y'(0) = 1.$$

(+1) $s^2Y(s) - 1 + 2sY(s) = e^{-s}$

$$(s^2+2s)Y(s) = e^{-s} + 1 \Rightarrow Y(s) = \frac{1}{s^2+2s} + \frac{e^{-s}}{s^2+2s}$$ (+1)

$$\frac{1}{s^2+2s} = \frac{1}{s(s+2)} = \frac{1}{2}\frac{(s+2)-s}{s(s+2)}$$

$$= \frac{1}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right)$$

(+1) $\mathcal{L}^{-1}\left\{\frac{1}{s^2+2s}\right\} = \frac{1}{2} - \frac{1}{2}e^{-2t}$ (+1)

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+2s}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+2s}\right\}\Big|_{t\to t-1} u_1(t)$$

$$= \boxed{\frac{1}{2} - \frac{1}{2}e^{-2t} + \left(\frac{1}{2} - \frac{1}{2}e^{-2t+2}\right)u_1(t)}$$ (+1)