NAME: Solutions

Georgia Tech, Fall 2015
Math 2552 (Sections F1 – F4)

Quiz 1

1. [5 points] Solve the ODE below and give an interval of validity for your solutions:

$$\frac{dy}{dx} = xy + 2x + y + 2.$$

2. [5 points] Solve the initial value problem below and give an interval of validity for your solutions:

$$y' + y^2\cos(2x) = 0;\ \ y(0) = 1.$$

① $\frac{dy}{dx} = (x+1)(y+2)$ (½ pt.) - factoring

$\frac{1}{y+2}dy = (x+1)\,dx$ [if $y \neq -2$] (½ pt.) - separating

$\int \frac{1}{y+2}dy = \int (x+1)\,dx$ (½ pt.) - integrating both sides

$\ln|y+2| = \frac{x^2}{2} + x + C$ (2 pts.) - antiderivatives (1 pt. each)

$|y+2| = e^{x^2/2 + x + C}$

$y+2 = Ce^{x^2/2 + x}$

(1 pt.) - explicit solution $\boxed{y = -2 + Ce^{x^2/2 + x}}$ $\boxed{x \in (-\infty, \infty) \text{ or } x \in \mathbb{R}}$ (½ pt.)

Remark: The constant solution $y = -2$ is represented in this formula by allowing $C = 0$

[(+½ pt) Bonus for explaining this]

② $\frac{dy}{dx} = -y^2\cos(2x)$

$-\frac{1}{y^2}dy = \cos(2x)\,dx$ (½ pt.) - separating

$\int -\frac{1}{y^2}dy = \int \cos(2x)\,dx$ (½ pt.) - integrating both sides

$\frac{1}{y} = \frac{1}{2}\sin(2x) + C$ (2 pts.) - antideriv. (1 pt. each)

I.C.: $y(0) = 1 \Rightarrow 1 = \frac{1}{2}\sin(0) + C$

$\Rightarrow \boxed{C = 1}$ (½ pt.) - finding C

$\frac{1}{y} = \frac{1}{2}\sin(2x) + 1$

$\boxed{y = \frac{1}{\frac{1}{2}\sin(2x) + 1}}$ or $\boxed{y = \frac{2}{\sin(2x) + 2}}$

(1 pt.) - explicit solution

(½ pt.) $\Rightarrow \boxed{x \in (-\infty, \infty) \text{ or } x \in \mathbb{R}}$ (because $\sin(2x)$ can never be -2)

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Quiz 1

1. [5 points] Solve the ODE below and give an interval of validity for your solution:

$$x\frac{dy}{dx} = 2(y-4).$$

2. [5 points] Solve the initial value problem below and give an interval of validity for your solution:

$$y' = -\frac{2xy^2}{1+x^2};\ y(0) = \frac{1}{2}.$$

① $\frac{1}{y-4}\,dy = \frac{2}{x}\,dx$ (½ pt.) - separating

$\int \frac{1}{y-4}\,dy = \int \frac{2}{x}\,dx$ [if $y \neq 4$] (½ pt.) - integrating both sides

$\ln|y-4| = 2\ln|x| + C$ (2 pts.) - antiderivatives (1 pt. each)

$$\left.\begin{aligned}|y-4| &= e^{2\ln|x|+C}\\ &= e^C\left(e^{\ln|x|}\right)^2\\ &= e^C x^2\\ y-4 &= \pm e^C x^2\\ y-4 &= C x^2\end{aligned}\right\}$$ (½ pt.) - simplification

(1 pt.) - explicit solution $\boxed{y = 4 + Cx^2}$ $\boxed{x \in (-\infty, \infty) \text{ or } x \in \mathbb{R}}$ (½ pt.)

Remark: The constant solution $y=4$ is represented in the formula by allowing $C=0$

[(+½ pt.) Bonus for explaining this]

② $\frac{dy}{dx} = -\frac{2xy^2}{1+x^2}$

$-\frac{1}{y^2}\,dy = \frac{2x}{1+x^2}\,dx$ (½ pt.) - separating

$\int -\frac{1}{y^2}\,dy = \int \frac{2x}{1+x^2}\,dx$ (½ pt.) - integrating both sides

$\frac{1}{y} = \ln(1+x^2) + C$ (2 pts.) - antideriv. (1 pt. each)

I.C.: $y(0) = \frac{1}{2}$

$2 = \ln(1) + C \Rightarrow \boxed{C = 2}$ (½ pt.) - finding C

$\frac{1}{y} = \ln(1+x^2) + 2$

$\boxed{y = \frac{1}{\ln(1+x^2)+2}}$ (1 pt.) - explicit solution

$\boxed{x \in (-\infty, \infty) \text{ or } x \in \mathbb{R}}$ (½ pt.)

($x \in \mathbb{R}$, because $\ln(1+x^2)+2 \geq 2$
$1+x^2 \geq 1$, $\forall x$
$\Rightarrow \ln(1+x^2) \geq 0$, $\forall x$)