Worksheet 7 Solutions

(1) $$\lim_{\substack{(x,y)\to(0,0)\\ y=kx}} \frac{y^2}{x^2+y^2} = \lim_{x\to 0} \frac{k^2x^2}{x^2+k^2x^2} = \frac{k^2}{1+k^2}$$

Limit DNE by Two-Path Test.

(2) $$\lim_{\substack{(x,y)\to(0,0)\\ y=kx}} \frac{|x|}{|x|+|y|} = \lim_{x\to 0} \frac{|x|}{|x|+|kx|} = \lim_{x\to 0} \frac{1}{1+|k|} = \frac{1}{1+|k|}$$

Limit DNE by Two-Path Test

(3) $$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} = \lim_{r\to 0} \frac{r^3(\cos^3\theta+\sin^3\theta)}{r^2(\cos^2\theta+\sin^2\theta)} = \lim_{r\to 0} r\underbrace{(\cos^3\theta+\sin^3\theta)}_{\text{bounded}} = \boxed{0}$$

(4) $$\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^2}$$

$$y=kx \Rightarrow \frac{kx^4}{x^6+k^2x^2} = \frac{kx^2}{x^4+k^2} \xrightarrow[x\to 0]{} 0$$

$$y=kx^3 \Rightarrow \frac{kx^6}{x^6+k^2x^6} = \frac{k}{1+k^2}$$

Limit DNE by Two-Path Test

(5) $$\lim_{(x,y)\to(0,0)} \frac{e^{-3y}\sin(-2x)}{-3x} = \lim_{(x,y)\to(0,0)} \underbrace{\frac{\sin(-2x)}{-2x}}_{\xrightarrow[x\to 0]{} 1} \cdot \frac{2e^{-3y}}{3} = \boxed{\frac{2}{3}}$$

$$\boxed{\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1}$$

(6) $$\lim_{(x,y)\to(3,4)} \frac{xy-3y-7x+21}{x-3} = \lim_{y\to 4}(y-7) = \boxed{-3}$$

$$xy-3y-7x+21 = y(x-3)-7(x-3) = (x-3)(y-7)$$

(7) $$\lim_{(x,y)\to(1,0)} \frac{\sqrt{x-4y}-1}{x-4y-1} = \lim_{(x,y)\to(1,0)} \frac{\cancel{\sqrt{x-4y}-1}}{(\cancel{\sqrt{x-4y}-1})(\sqrt{x-4y}+1)} = \lim_{(x,y)\to(1,0)} \frac{1}{\sqrt{x-4y}+1} = \boxed{\frac{1}{2}}$$

(8) $$\lim_{(x,y)\to(0,0)} \frac{\sin(x^{10}+y^{10})}{x^{10}+y^{10}} = \boxed{1}.$$

(9) $$\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3-y^3}{x^2+y^2}\right) = \lim_{r\to 0} \cos\left(\frac{r^3(\cos^3\theta-\sin^3\theta)}{r^2}\right) = \lim_{r\to 0} \cos\left(\overbrace{r\underbrace{(\cos^3\theta-\sin^3\theta)}_{\text{bounded}}}^{0 \uparrow r\to 0}\right) = \boxed{1}$$