1. Find:

$$\int_C 2xy\,dx + 4y\,dy - yz\,dz$$

where $C$ is the curve:

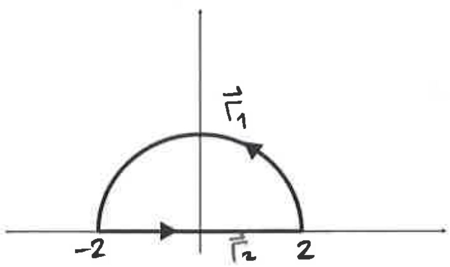
$$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}, \quad 0 \le t \le 1.$$

2. Find the circulation and flux of the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ around the closed semicircular path that consists of the half-circle

$$\mathbf{r}_1(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j}, \quad 0 \le t \le \pi,$$

followed by the line segment

$$\mathbf{r}_2(t) = t\mathbf{i}, \quad -2 \le t \le 2. \ .$$




(4 pts.)

(1) $x = t$ ; $dx = dt$
$y = t^2$ ; $dy = 2t\,dt$
$z = t$ ; $dz = dt$

$$\int_C 2xy\,dx + 4y\,dy - yz\,dz =$$

$$= \int_0^1 \left(2t^3 + 4t^2 \cdot 2t - t^3\right)dt = \int_0^1 9t^3\,dt$$

$$= \frac{9t^4}{4}\Big|_0^1 = 9/4$$

(3 pts)

(2) Circulation: $\int_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_2} \vec{F}\cdot d\vec{r} = 0 + 0 = \boxed{0}$

$$\int_{C_1} \vec{F}\cdot d\vec{r} = \int_{C_1} M\,dx + N\,dy = \int_0^{\pi} 2\cos t(-2\sin t) + (2\sin t)(2\cos t)\,dt = \boxed{0}$$

$x = 2\cos t$ ; $dx = -2\sin t\,dt$
$y = 2\sin t$ ; $dy = +2\cos t\,dt$
$M = x$ ; $N = y$

$$\int_{C_2} \vec{F}\cdot d\vec{r} = \int_{C_2} M\,dx + N\,dy = \int_{-2}^{2} t\,dt = \frac{t^2}{2}\Big|_{-2}^{2} = \boxed{0}$$

$x = t$ ; $dx = dt$
$y = 0$ ; $dy = 0$

(3pts)

Flux: $\int_{C_1} \vec{F}\cdot\vec{n}\,ds = \int_{C_1} M\,dy - N\,dx$

$$= \int_0^{\pi} 2\cos t\,(2\cos t) - (2\sin t)(-2\sin t)\,dt$$

$$= \int_0^{\pi} 4(\cos^2 t + \sin^2 t)\,dt$$

$$= 4\pi$$

$$\int_{C_2} \vec{F}\cdot\vec{n}\,ds = \int_{C_2} M\,dy - N\,dx$$

$$= \int_{-2}^{2} (t\cdot 0 - 0)\,dt = 0$$

$$\oint_C \vec{F}\cdot\vec{n}\,ds = \boxed{4\pi}$$

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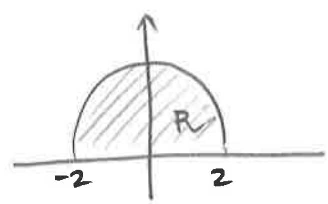
OR, using Green's Theorem:




$M = x;\ N = y$

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dA$$

$$= \iint_R (0 - 0)\,dA = \boxed{0}$$

$$\oint_C \vec{F}\cdot\vec{n}\,ds = \iint_R \left(\frac{\partial M}{\partial x} + \frac{\partial N}{\partial y}\right) dA$$

$$= \iint_R (1 + 1)\,dA = 2\,\text{Area}(R) = 2\cdot\frac{\pi\cdot 2^2}{2} = \boxed{4\pi}$$