Name: ______________________________

May 5th, 2016.
Math 2551; Sections L1, L2, L3.
Georgia Institute of Technology
Final Exam

I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech community. By signing my name below I pledge that I have neither given nor received help on this exam.

Pledged: ______________________________

Problem	Possible Score	Earned Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
Total	140	

Remember that you must SHOW YOUR WORK to receive credit!

Good luck!

1. (a). Write parametric equations for the line joining the points $(0, 8, 0)$ and $(8, 0, 0)$.

(b). Let C be the line segment from $(0, 8, 0)$ to $(8, 0, 0)$. Compute the line integral:

$$\int_C (x + y)\, ds.$$

2. Given that for a curve $\mathbf{r}(t)$:

$$\frac{d\mathbf{r}}{dt} = \frac{1}{\sqrt{t}}\,\mathbf{i} + \sin(t)\,e^{\cos(t)}\,\mathbf{j} + t\,\mathbf{k},$$

and that:

$$\mathbf{r}(0) = \langle 1, 0, 3 \rangle\,,$$

find $\mathbf{r}(t)$.

3. Compute

$$\int_{\pi/4}^{\pi/2} \int_{0}^{\frac{2}{\cos\theta + \sin\theta}} r^2 \cos\theta \, dr \, d\theta.$$

4. (a). Find

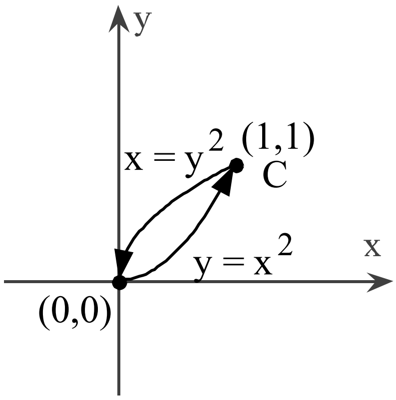
$$\oint_C y^2\,dx + 3xy\,dy,$$

where C is the closed, positively oriented curve consisting of the upper half of the unit circle $x^2 + y^2 = 1$ $(y > 0)$, and the line segment joining $(-1, 0)$ and $(1, 0)$.

(b). Find the outward flux of the field

$$\mathbf{F}(x, y) = \left\langle 2xy + y^2,\ 2x - y \right\rangle,$$

across the curve C pictured below.



5. Find the following limits:

a). $$\lim_{(x,y)\to(0,0)} \frac{e^{-2y}\sin(3x)}{5x}$$

b). $$\lim_{(x,y)\to(2,8)} \frac{xy-2y-7x+14}{x-2}.$$

c). Show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x}{\sqrt{x^2+y^2}}.$$

6. Find the minimum and maximum value of $f(x, y) = x^2 + y^2$ subject to the constraint

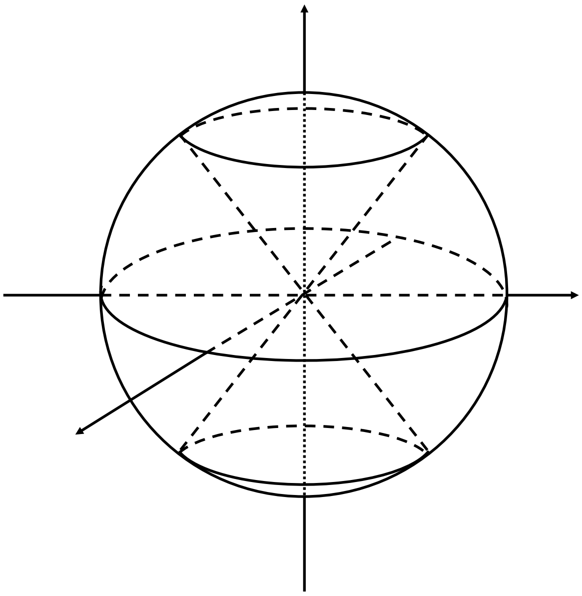
$$x^2 - 4x + y^2 - 2y = 0.$$

Indicate the points where these extreme values occur.

7. (a). Let x be a real number. What is $\sqrt{x^2}$?

$$\sqrt{x^2} =$$

(b). Let D be the solid that lies *inside* the sphere $x^2+y^2+z^2=4$ and *outside* the cone $z^2=3(x^2+y^2)$. Use *spherical* coordinates to set up the triple integral that gives the volume of D. You do not need to compute the volume.



8. Find the outward flux of the field:

$$\mathbf{F}(x, y, z) = \left\langle 3x^3 + 3xy^2,\ 2y^3 + e^y \sin z,\ 3z^3 + e^y \cos z \right\rangle,$$

across the boundary of the region D given by $1 \leq x^2 + y^2 + z^2 \leq 2$.

9. Consider the curve:

$$\mathbf{r}(t) = \left\langle 2\cos t,\ \ 2\sin t,\ \ \sqrt{5}t \right\rangle.$$

(a). Find the arc length parameter along this curve, taking $(2, 0, 0)$ for the initial point.

(b). Find the length of the portion of this curve with $0 \le t \le \pi/6$.

(c). Find the point on this curve that is at distance π units along the curve from $(2, 0, 0)$ in the direction of increasing arc length.

10. Consider the surface S given by the parametrization:

$$\mathbf{r}(r,\theta) = \langle r\cos\theta,\ r\sin\theta,\ r\rangle\,;\ r \geq 0;\ 0 \leq \theta \leq 2\pi.$$

(a). Find a Cartesian equation for the surface S and sketch it.

(b). Find the equation of the plane tangent to the surface S at the point $P_0(-1, \sqrt{3}, 2)$ corresponding to $(r, \theta) = (2, 2\pi/3)$.

11. Find all the critical points of the function:

$$f(x, y) = x^3 + y^3 + 3x^2 - 6y^2 - 1,$$

and classify each one as a local minimum, a local maximum, or a saddle point.

12. Consider the curve:

$$\mathbf{r}(t) = \langle 6\sin t,\ \ 6\cos t,\ \ 8t\rangle\,,$$

and its unit tangent and unit normal vectors:

$$\mathbf{T} = \left\langle \frac{3}{5}\cos t,\ \ -\frac{3}{5}\sin t,\ \ \frac{4}{5}\right\rangle,$$

$$\mathbf{N} = \langle -\sin t,\ \ -\cos t,\ \ 0\rangle\,.$$

a). Find the unit binormal vector **B**.

b). Find the torsion τ along this curve.

13. Consider the lines:

$$\text{L1:} \quad x = -1 + 2t, \quad y = 2 + 3t, \quad z = 1 - 2t;$$

$$\text{L2:} \quad x = 1 - 4s, \quad y = 1 + 2s, \quad z = 2 - 2s.$$

(a). Find the point of intersection of these lines.

(b). Find an equation of the plane determined by the two lines.

14. Prove that:

$$\int_{-\infty}^{\infty} e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}},$$

where a is any positive real number.