

Angle θ ($0 \leq \theta \leq \pi$) between vectors \mathbf{u} and \mathbf{v} :

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}; \quad \sin \theta = \frac{|\mathbf{u} \times \mathbf{v}|}{|\mathbf{u}||\mathbf{v}|}.$$

Vector Projection of \mathbf{u} onto $\mathbf{v} \neq 0$:

$$\text{Proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2} \right) \mathbf{v} = |\mathbf{u} \cos \theta| \frac{\mathbf{v}}{|\mathbf{v}|}.$$

Distance from a point S to a line L going through P and parallel to \mathbf{v} :

$$d = \frac{|\vec{PS} \times \mathbf{v}|}{|\mathbf{v}|}$$

Length of a smooth curve $C: \mathbf{r}(t)$, traced exactly once as $a \leq t \leq b$:

$$L = \int_a^b |\mathbf{v}(t)| dt.$$

Arclength parameter:

$$s(t) = \int_{t_0}^t |\mathbf{v}(\tau)| d\tau.$$

TNB Frame:

$$\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}; \quad \mathbf{N} = \frac{d\mathbf{T}/ds}{\kappa} = \frac{d\mathbf{T}/dt}{|d\mathbf{T}/dt|}; \quad \mathbf{B} = \mathbf{T} \times \mathbf{N}.$$

Curvature:

$$\kappa = \left| \frac{d\mathbf{T}}{ds} \right| = \frac{1}{|\mathbf{v}|} \left| \frac{d\mathbf{T}}{dt} \right| = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3}.$$

Tangential and Normal Components of Acceleration:

$$\mathbf{a} = a_T \mathbf{T} + a_N \mathbf{N};$$

$$a_T = \frac{d^2 s}{dt^2} = \frac{d}{dt} |\mathbf{v}(t)|;$$

$$a_N = \kappa \left(\frac{ds}{dt} \right)^2 = \kappa |\mathbf{v}(t)|^2 = \sqrt{|\mathbf{a}|^2 - a_T^2}.$$

Torsion:

$$\tau = -\frac{d\mathbf{B}}{ds} \cdot \mathbf{N} = -\frac{1}{|\mathbf{v}|} \frac{d\mathbf{B}}{dt} \cdot \mathbf{N}.$$
