Quiz 7 - Solutions.

1 $\boxed{\sum_{n=1}^{\infty} \frac{n}{n^2+2}}$ (7 pts.)

a). Divergence Test?

(2pts.) $a_n = \frac{n}{n^2+2}$; $\lim_{n\to\infty} \frac{n}{n^2+2} = 0$ => <u>No conclusion.</u>

b). Comparison Tests?

(2pts.) * <u>Limit Comparison</u>: $b_n = \frac{1}{n}$; $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^2}{n^2+2} = 1$

=> $\sum_{n=1}^{\infty} a_n$ <u>diverges</u> because $\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series).

OR

* <u>Comparison</u>: $a_n = \frac{n}{n^2+2} < \frac{n}{n^2} = \frac{1}{n}$ (No conclusion)

Try: $a_n = \frac{n}{n^2+2} > \frac{n}{n^2+2n} = \frac{1}{n+2}$

=> $\sum_{n=1}^{\infty} a_n \geq \sum_{n=1}^{\infty} \frac{1}{n+2} = \infty$ (harmonic series) => <u>Divergent</u>.

c). <u>Integral Test</u>?

(3pts.) $f(x) = \frac{x}{x^2+2}$ is positive, decreasing & continuous on $[1, \infty)$

$\int_1^\infty \frac{x}{x^2+2}\,dx = \frac{1}{2}\ln(x^2+2)\Big|_1^\infty = \infty$ => $\sum_{n=1}^{\infty} a_n$ <u>diverges</u>.

2 $\boxed{\sum_{n=1}^{\infty} (\cos(19))^n} = \sum_{n=1}^{\infty} \cos(19)\cdot(\cos(19))^{n-1}$

(3 pts.)

$= \boxed{\frac{\cos(19)}{1-\cos(19)}}$. (1pt.)

Geometric Series w/
$a = \cos(19)$; $r = \cos(19)$
$|r| < 1$ => <u>convergent</u>
to $\frac{a}{1-r}$

(2pts.)