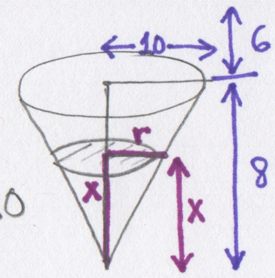
1).

$\rho = 80$



$\frac{r}{10} = \frac{x}{8}$

$r = \frac{5x}{4}$ (1pt.)

$$W = \underbrace{\int_0^8}_{\text{bounds (2pts.)}} \pi \underbrace{\frac{25x^2}{16}}_{(1pt.)} \cdot \underbrace{80}_{(1pt.)} \cdot \underbrace{(14-x)}_{(2pts.)} dx$$

$$= \pi \cdot \frac{25}{16} \cdot 80 \int_0^8 x^2(14-x)\,dx$$

$$\int_0^8 (14x^2 - x^3)\,dx$$

$$= \pi \cdot \frac{25}{16} \cdot 80 \left(14x^3\frac{1}{3} - \frac{x^4}{4}\right)\Big|_0^8$$ (2pts)

$$= \pi \cdot \frac{25}{16} \cdot 80 \left(\frac{14}{3}8^3 - \frac{1}{4}8^4\right)$$ (1pt.)

3). a). $f(x) = x^{\sin(3x)}$ (2pts each half)

$$f'(x) = \sin(3x)\,x^{\sin(3x)-1} + x^{\sin(3x)} \cdot \ln x \cdot 3\cos(3x)$$

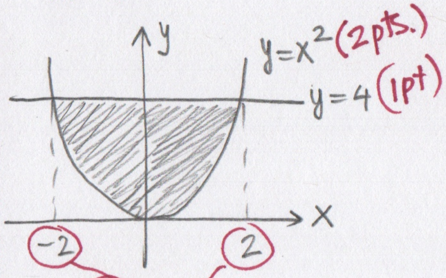
b). $f(x) = \log_3(\tan(5x)) = \frac{\ln(\tan 5x)}{\ln 3}$

$$f'(x) = \frac{1}{\ln 3}\,\frac{1}{\tan(5x)}\sec^2(5x) \cdot 5$$ (2pts)

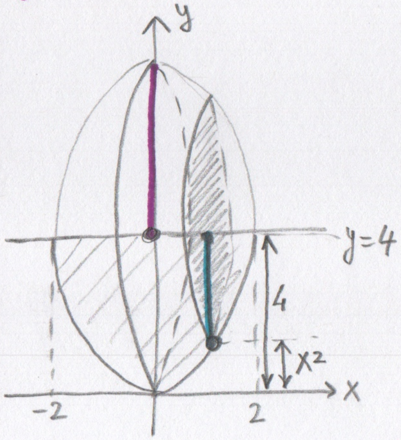
c). $f(x) = e^{\ln(\cos x)} = \cos x$

$$f'(x) = -\sin x$$ (2pts)

2). a).



b).

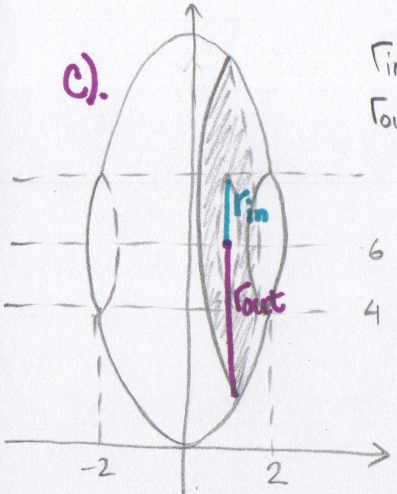


$$V = \int_{-2}^{2} \pi(4-x^2)^2\,dx$$ (3pts)

c).

$r_{in} = 2$

$r_{out} = 6 - x^2$



(3pts.)

$$V = \int_{-2}^{2} \left(\pi(6-x^2)^2 - \pi \cdot 4\right) dx$$

4). $\int_{-\infty}^{0} xe^x\,dx$

$$= \lim_{t \to -\infty} \int_t^0 xe^x\,dx$$ (1pt.)

$$\int_t^0 xe^x\,dx = xe^x\Big|_t^0 - \int_t^0 e^x\,dx$$ (1pt.)

$u = x \quad du = dx$

$dv = e^x dx \quad v = e^x$ (2pts)

$$= -te^t - e^x\Big|_t^0$$

$$= \boxed{-te^t - 1 + e^t}$$ (2pts.)

$$\lim_{t \to -\infty} te^t \underset{(0\cdot\infty)}{=} \lim_{t \to -\infty} \frac{t}{e^{-t}} = \frac{\infty}{\infty}$$ l'Hospital

$$= \lim_{t \to -\infty} \frac{1}{-e^{-t}} = \boxed{0}$$ (3pts.)

$\Rightarrow$

$$\int_{-\infty}^{0} xe^x\,dx = \lim_{t \to -\infty}\left(-te^t - 1 + e^t\right)$$

$$= 0 - 1 + 0 = \boxed{-1}$$ (1pt.)

5). $\int \sin^5 x \cos^2 x\, dx$

$= \int \sin x\,(1-\cos^2 x)^2 \cos^2 x\, dx$

$u = \cos x,\ du = -\sin x\, dx$ } (5pts) Setup

$= -\int (1-u^2)^2 u^2\, du$ (2pts.)

$= -\int (1-2u^2+u^4)u^2\, du$

$= \int (2u^4 - u^2 - u^6)\, du$ (1pt.)

$= 2\frac{u^5}{5} - \frac{u^3}{3} - \frac{u^7}{7} + C$ (1pt.)

$= \boxed{\frac{2}{5}\cos^5 x - \frac{1}{3}\cos^3 x - \frac{1}{7}\cos^7 x + C}$ (1pt.)

6). (2 pts.) each

1. $\int \frac{1}{x+2}\, dx = \boxed{\ln|x+2| + C}$

2. $\int \frac{x}{x+2}\, dx = \int \frac{x+2-2}{x+2}\, dx$

$= \int \left(1 - \frac{2}{x+2}\right) dx$

$= \boxed{x - 2\ln|x+2| + C}$

3. $\int \frac{x^2}{x+2}\, dx = \int \frac{x^2+2x-2x}{x+2}\, dx$

$= \int \frac{x(x+2) - 2x - 4 + 4}{x+2}\, dx$

(long division)

$= \int x - \frac{2(x+2)-4}{x+2}\, dx$

$= \int x - 2 + \frac{4}{x+2}\, dx$

$= \boxed{\frac{1}{2}x^2 - 2x + 4\ln|x+2| + C}$

4. $\int \frac{1}{x^2+2}\, dx = \boxed{\frac{1}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right) + C}$

5. $\int \frac{x}{x^2+2}\, dx = \boxed{\frac{1}{2}\ln(x^2+2) + C}$

6. $\int \frac{x^2}{x^2+2}\, dx = \int \frac{x^2+2-2}{x^2+2}\, dx$

$= \int 1 - \frac{2}{x^2+2}\, dx$

$= \boxed{x - \sqrt{2}\arctan\left(\frac{x}{\sqrt{2}}\right) + C}$