Quiz 5 - Solutions

① $\int x \ln(3x)\,dx$ — By parts:

$u = \ln(3x)$, $du = \frac{1}{x}dx$

$dv = x\,dx$, $v = \frac{1}{2}x^2$

2 pts.

$= \frac{1}{2}x^2\ln(3x) - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$ (1 pt.)

$= \frac{1}{2}x^2\ln(3x) - \frac{1}{2}\int x\,dx$

$= \boxed{\frac{1}{2}x^2\ln(3x) - \frac{1}{4}x^2 + C}$ (½ pt.)

② $\int \arctan(2x)\,dx$ — By parts:

$u = \arctan(2x)$, $du = \frac{2}{1+4x^2}$

$dv = dx$, $v = x$

2 pts.

$= x\arctan(2x) - \int \frac{2x}{1+4x^2}\,dx$ (1 pt.)

$= \boxed{x\arctan(2x) - \frac{1}{4}\ln(1+4x^2) + C}$ (½ pt.)

③ $\int \frac{1}{x^2\sqrt{x^2-25}}\,dx$

$x = 5\sec\theta$; $\theta \in [0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$ → 1 pt.

$dx = 5\sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-25} = 5\tan\theta$

(1 pt.) $= \int \frac{1}{25\sec^2\theta \cdot 5\tan\theta} 5\sec\theta\tan\theta\,d\theta$

$= \frac{1}{25}\int \frac{1}{\sec\theta}\,d\theta$

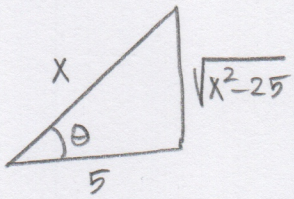
$= \frac{1}{25}\int \cos\theta\,d\theta$

$= \frac{1}{25}\sin\theta + C$

(½ pt.) $= \boxed{\frac{1}{25}\frac{\sqrt{x^2-25}}{x} + C}$

$\sec\theta = \frac{x}{5}$

$\cos\theta = \frac{5}{x}$



$\boxed{\sin\theta = \frac{\sqrt{x^2-25}}{x}}$ (½ pt.)