Name: *Solutions*

Clear your desk of everything except pens, pencils and erasers. **Show all your work.** If you have a question raise your hand and I will come to you.

**1.** [5 points] Solve the initial value problem below. You may leave the answer in implicit form.

$$\left(\frac{1}{e^y}+1\right)\sin x = (1+\cos x)\frac{dy}{dx}; \quad y(0)=0.$$

$$\frac{\sin x}{1+\cos x}\,dx = \frac{1}{\frac{1}{e^y}+1}\,dy$$ Separating: 1pt.

$$\int \frac{\sin x}{1+\cos x}\,dx = \int \frac{e^y}{1+e^y}\,dy$$

$$-\ln(1+\cos x) = \ln(1+e^y)+C$$ Antiderivatives: 2pts.

$$e^{-\ln(1+\cos x)} = Ce^{\ln(1+e^y)}$$

$$\frac{1}{1+\cos x} = C(1+e^y)$$

I.C.: $\frac{1}{1+\cos 0} = C(1+e^{y(0)})$

$\frac{1}{2} = C(1+1)$ $\quad C=\frac{1}{4}$ Constant: 1.5pts.

$\Rightarrow$ $\boxed{\frac{1}{1+\cos x} = \frac{1}{4}(1+e^y)}$ or $\boxed{4=(1+e^y)(1+\cos x)}$ Final answer: 0.5pts.

**2.** [5 points] Suppose that the voltage $V(t)$ of electricity at time $t$ (in seconds) is draining from a capacitor at a rate that is proportional to its value. That is, V(t) satisfies the differential equation

$$V'(t) = -kV(t),$$

where $k>0$ is the constant of proportionality. If $k=\frac{1}{14}$, how long will it take the voltage to drop to 20 percent of its original value?

1pt. $V(t)=V(0)e^{-kt} = V(0)e^{-\frac{1}{14}t}$

1pt. $V(t)=\frac{1}{5}V(0) \Rightarrow V(0)e^{-\frac{1}{14}t} = \frac{1}{5}V(0)$ $\Rightarrow 5=e^{t/14}$ 1pt.

$\Rightarrow \ln 5 = t/14$ 1pt.

$\Rightarrow \boxed{t=14\ln 5}$ 1pt.