MONDAY 10/9

Improper Integral

1. $\int_{-\infty}^{-1} \frac{1}{x}\,dx = \ln|x|\Big|_{-\infty}^{-1} = 0 - \infty = \boxed{-\infty}$

DIVERGENT.

Substitution

2. $\int \sqrt{2x+1}\,dx$

$u = 2x+1$
$du = 2dx$

$= \int \sqrt{u}\cdot\frac{1}{2}du = \frac{2}{3}u^{3/2}\cdot\frac{1}{2} + C = \boxed{\frac{1}{3}(2x+1)^{3/2} + C}$

Partial Fractions

3. $\int \frac{3x-9}{(x-1)(x+2)^2}\,dx$

$= \int \frac{-2/3}{x-1} + \frac{2/3}{x+2} + \frac{5}{(x+2)^2}\,dx = \boxed{-\frac{2}{3}\ln|x-1| + \frac{2}{3}\ln|x+2| - \frac{5}{x+2} + C}$

$\frac{3x-9}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

$3x-9 = A(x+2)^2 + B(x-1)(x+2) + C(x-1)$

$x=1: \; -6 = 9A \quad \boxed{A = -2/3}$

$x=-2: \; -15 = -3C \quad \boxed{C=5}$

$x=0: \; -9 = -\frac{8}{3} - 2B - 5$

$-4 + \frac{8}{3} = -2B$

$2B = \frac{4}{3} \quad \boxed{B = \frac{2}{3}}$

By Parts

4. $\int_1^4 \sqrt{x}\ln x\,dx$

$u = \ln x \qquad du = \frac{1}{x}dx$
$dv = \sqrt{x}dx \qquad v = \frac{2}{3}x^{3/2}$

$= \frac{2}{3}x^{3/2}\ln x\Big|_1^4 - \int_1^4 \frac{2}{3}x^{3/2}\frac{1}{x}dx$

$= \frac{2}{3}\cdot 8\ln 4 - \frac{4}{9}x^{3/2}\Big|_1^4 = \frac{16}{3}\ln 4 - \frac{32}{9} + \frac{4}{9}$

$= \boxed{\frac{16}{3}\ln 4 - \frac{28}{9}}$

Trig Substitution

5. $\int \frac{dx}{\sqrt{x^2+4x+13}}$

$= \int \frac{dx}{\sqrt{(x+2)^2+9}}$

$x+2 = 3\tan\theta, \; \theta \in (-\pi/2, \pi/2)$
$dx = 3\sec^2\theta$
$\sqrt{\cdots} = 3\sec\theta$

$= \int \frac{3\sec^2\theta}{3\sec\theta}d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{x^2+4x+13}}{3} + \frac{x+2}{3}\right| + C$

$= \boxed{\ln\left|\sqrt{x^2+4x+13} + x+2\right| + C}$

$\tan\theta = \frac{x+2}{3}$