Partial Fractions

1. $\int \frac{x^3+1}{x^2-4}\,dx = \int \frac{x^3-4x+4x+1}{x^2-4}\,dx$

$= \int \frac{x(x^2-4)+4x+1}{x^2-4}\,dx$

$= \int x + \frac{4x+1}{x^2-4}\,dx$

$= \boxed{\frac{1}{2}x^2 + \frac{9}{4}\ln|x-2| + \frac{7}{4}\ln|x+2| + C}$

$\frac{4x+1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2}$

$4x+1 = A(x+2)+B(x-2)$

$x=2: 9=4A \quad \boxed{A=9/4}$

$x=-2: -7=-4B \quad \boxed{B=7/4}$

Trig Integral

2. $\int \sin^3 x \cdot \cos^{5/2} x\,dx$

$= \int \sin x \cdot (1-\cos^2 x)\cdot \cos^{5/2} x\,dx \qquad u=\cos x,\ du=-\sin x\,dx$

$= -\int (1-u^2)u^{5/2}\,du = \int \left(u^{9/2} - u^{5/2}\right)du$

$= \frac{2}{11}u^{11/2} - \frac{2}{7}u^{7/2} + C$

$= \boxed{\frac{2}{11}(\cos x)^{11/2} - \frac{2}{7}(\cos x)^{7/2} + C}$

SUNDAY 10/9

Substitution

3. $\int \frac{x}{1+x^4}\,dx \qquad u=x^2,\ du=2x\,dx$

$= \frac{1}{2}\int \frac{1}{1+u^2}\,du = \frac{1}{2}\arctan(u)+C = \boxed{\frac{1}{2}\arctan(x^2)+C}$

Method 1: Substitution

4. $\int x^5\sqrt{1+x^2}\,dx$

$u=x^2+1$, $du=2x\,dx \Rightarrow x\,dx=\frac{1}{2}du$

$\Rightarrow x^2=u-1 \Rightarrow x^4=(u-1)^2$

$= \int x\cdot x^4\sqrt{1+x^2}\,dx$

$= \int (u-1)^2\sqrt{u}\cdot\frac{1}{2}du = \frac{1}{2}\int (u^2-2u+1)\sqrt{u}\,du = \frac{1}{2}\int (u^{5/2}-2u^{3/2}+\sqrt{u})\,du$

$= \frac{1}{2}\left(\frac{2}{7}u^{7/2} - 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right)+C$

$= \boxed{\frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C}$

Method 2: Trig Substitution

$x=\tan\theta,\ \theta\in(-\pi/2,\pi/2)$

$\int \tan^5\theta\cdot\sec\theta\cdot\sec^2\theta\,d\theta = \int \tan^5\theta\cdot\sec^3\theta\,d\theta$

$= \int \tan^4\theta\cdot\sec^2\theta\cdot(\tan\theta\sec\theta)\,d\theta \qquad u=\sec\theta$

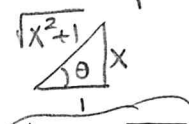
$= \int (u^2-1)^2u^2\,du = \int (u^4-2u^2+1)u^2\,du = \int (u^6-2u^4+u^2)\,du$

$= \frac{1}{7}u^7 - \frac{2}{5}u^5 + \frac{1}{3}u^3 + C$

$= \frac{1}{7}(\sec\theta)^7 - \frac{2}{5}(\sec\theta)^5 + \frac{1}{3}(\sec\theta)^3 + C$

$= \boxed{\frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C}$

$\tan\theta = \frac{x}{1}$



$\sec\theta = \sqrt{x^2+1}$

By Parts

5. $\int x\ln x\,dx$

$u=\ln x \quad du=\frac{1}{x}dx$

$dv=x\,dx \quad v=\frac{1}{2}x^2$

$= \frac{1}{2}x^2\ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}\,dx$

$= \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C}$