SATURDAY 10/7

Substitution + By Parts

1. $$\int \frac{\ln(\ln x)\cdot \ln x}{x}\,dx \qquad y=\ln x \quad dy=\frac{1}{x}dx$$

$$=\int \ln(y)\cdot y\,dy \quad = \frac{1}{2}y^2\ln y - \int \frac{1}{2}y^2\frac{1}{y}dy$$

$u=\ln y \quad du = 1/y\,dy$

$dv = y \quad v=\frac{1}{2}y^2$

$$=\frac{1}{2}y^2\ln y - \frac{1}{4}y^2 + C$$

$$=\boxed{\frac{1}{2}\ln^2 x\cdot \ln(\ln x) - \frac{1}{4}\ln^2 x + C}$$

Substitution

2. $$\int \frac{x}{\sqrt{1-4x^2}}\,dx \qquad u = 1-4x^2 \quad du=-8x\,dx \;:\; x\,dx = -\frac{1}{8}du$$

$$=\int \frac{1}{\sqrt{u}}\cdot\frac{-1}{8}du = -\frac{1}{4}\int\frac{du}{2\sqrt{u}} = -\frac{1}{4}\sqrt{u}+C$$

$$=\boxed{-\frac{1}{4}\sqrt{1-4x^2}+C}$$

Improper Integral: Problem @ $x=1$!

3. $$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$$

$$=\lim_{t\to 1^-}\left(\int_0^t \frac{1}{\sqrt{1-x^2}}dx\right) = \lim_{t\to 1^-}\left(\arcsin(x)\Big|_0^t\right)$$

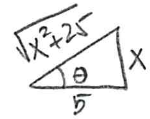
$$=\lim_{t\to 1^-}\left(\arcsin(t)-\arcsin(0)\right) = \boxed{\frac{\pi}{2}}$$

Trig Sub. w/ $x=5\tan\theta$, $dx = 5\sec^2\theta\,d\theta$, $\theta\in(-\pi/2, \pi/2)$

4. $$\int \frac{dx}{\sqrt{25+x^2}}$$

$$=\int \frac{1}{5\sec\theta}5\sec^2\theta\,d\theta = \int \sec\theta\,d\theta = \ln|\sec\theta+\tan\theta|$$

$\tan\theta = \frac{x}{5}$



$$=\ln\left|\frac{\sqrt{x^2+25}}{5}+\frac{x}{5}\right|+C$$

$$=\boxed{\ln\left|\sqrt{x^2+25}+x\right|+C}$$

Partial Fractions

5. $$\int \frac{18}{(x+3)(x^2+9)}\,dx$$

$$\frac{18}{(x+3)(x^2+9)} = \frac{A}{x+3}+\frac{Bx+C}{x^2+9}$$

$$18 = A(x^2+9)+(Bx+C)(x+3)$$

$x=-3:\; 18=18A \quad \boxed{A=1}$

$x=0:\; 18=9+3C \quad \boxed{C=3}$

$x=1:\; 18 = 10+(B+3)\cdot 4$

$2 = B+3 \quad \boxed{B=-1}$

Split — Substitution — arctan

$$=\int\left(\frac{1}{x+3}+\frac{-x+3}{x^2+9}\right)dx = \ln|x+3| - \int\frac{x}{x^2+9}dx + 3\int\frac{1}{x^2+9}dx$$

$$=\boxed{\ln|x+3| - \frac{1}{2}\ln(x^2+9)+\arctan\left(\frac{x}{3}\right)+C}$$