FRIDAY 10/6

Trig Substitution

1. $\displaystyle\int \frac{dx}{x^3\sqrt{x^2-4}}$

$x = 2\sec\theta$, $\theta \in [0, \pi/2) \cup [\pi, 3\pi/2)$

$dx = 2\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-4} = 2\tan\theta$

$$= \int \frac{\cancel{2}\cancel{\sec\theta}\cancel{\tan\theta}}{8\sec^{\cancel{3}}\theta \cdot \cancel{2}\cancel{\tan\theta}}\, d\theta = \frac{1}{8}\int \frac{1}{\sec^2\theta}\, d\theta = \frac{1}{8}\int \cos^2\theta\, d\theta$$

Half-angle formula!

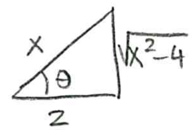
$$= \frac{1}{8}\int \frac{1}{2}(1+\cos(2\theta))\, d\theta = \frac{1}{16}\Big(\theta + \frac{1}{2}\underbrace{\sin(2\theta)}_{2\sin\theta\cos\theta}\Big)$$

$$= \frac{1}{16}(\theta + \sin\theta\cos\theta)$$

$$= \boxed{\frac{1}{16}\left(\sec^{-1}\left(\frac{x}{2}\right) + \frac{2\sqrt{x^2-4}}{x^2}\right) + C}$$

$\sec\theta = \frac{x}{2}$

$\cos\theta = \frac{2}{x}$



Partial Fractions

2. $\displaystyle\int \frac{x^2+2}{(x-1)(2x-8)(x+2)}\, dx$

$$= \boxed{-\frac{1}{6}\ln|x-1| + \ln|2x-8| + \frac{1}{6}\ln|x+2| + C}$$

$$\frac{x^2+2}{(x-1)(2x-8)(x+2)} = \frac{A}{x-1} + \frac{B}{2x-8} + \frac{C}{x+2}$$

$$x^2+2 = A(2x-8)(x+2) + B(x-1)(x+2) + C(x-1)(2x-8)$$

$x=4$: $18 = 18B$ $\boxed{B=1}$

$x=-2$: $6 = 36C$ $\boxed{C = 1/6}$

$x=1$: $3 = -18A$ $\boxed{A = -1/6}$

$-3\cdot -12$

$-6\cdot 3$

Improper Integral

3. $\displaystyle\int_2^\infty \frac{1}{x^2}\, dx$

$$= -\frac{1}{x}\Big|_2^\infty$$

$$= 0 + \frac{1}{2}$$

$$= \boxed{1/2}$$

$$= \lim_{t\to\infty} \int_2^t \frac{1}{x^2}\, dx$$

$$= \lim_{t\to\infty}\left(-\frac{1}{x}\Big|_2^t\right)$$

$$= \lim_{t\to\infty}\left(-\frac{1}{t} + \frac{1}{2}\right) = 0 + \frac{1}{2} = \boxed{\frac{1}{2}}.$$

Substitution

4. $\displaystyle\int x^3\cos(x^4+2)\, dx$

$$= \boxed{\frac{1}{4}\sin(x^4+2) + C}$$

$u = x^4+2$

$du = 4x^3\, dx.$

By Parts

5. $\displaystyle\int x\sec^2 x\, dx$

$u = x$ $\quad du = dx$

$dv = \sec^2 x\, dx$ $\quad v = \tan x$

$$= x\tan x - \int \tan x\, dx$$

$$= \boxed{x\tan x - \ln|\sec x| + C}$$