Thursday 10/5

Trig Integral + Substitution

1. $\int \sin^5 x \cos^3 x\, dx = \int \sin^5 x(1-\sin^2 x)\cos x\, dx$

$u = \sin x;\ du = \cos x\, dx$

$= \int u^5(1-u^2)\,du = \int (u^5 - u^7)\,du$

$= \frac{1}{6}u^6 - \frac{1}{8}u^8 + C$

$= \boxed{\frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C}$

Also OK: (just messier) $\int \sin x(1-\cos^2 x)^2\cos^3 x\, dx$

$u = \cos x;\ du = -\sin x\, dx$

$= -\int (1-u^2)^2 u^3\, du$

$= -\int (1-2u^2+u^4)u^3\, du$

$= -\int (u^3 - 2u^5 + u^7)\,du = -\left(\frac{1}{4}u^4 - \frac{2}{6}u^6 + \frac{1}{8}u^8\right) = \boxed{\frac{1}{3}\cos^6 x - \frac{1}{4}\cos^4 x - \frac{1}{8}\cos^8 x + C}$

Partial Fractions

2. $\int \frac{dx}{x^2-7x+10} = \int \left(-\frac{1}{3}\frac{1}{x-2} + \frac{1}{3}\frac{1}{x-5}\right)dx$

$\frac{1}{x^2-7x+10} = \frac{1}{(x-2)(x-5)} = \frac{A}{x-2} + \frac{B}{x-5}$

$1 = A(x-5) + B(x-2)$

$x=5:\ 1 = 3B \quad \boxed{B = 1/3}$

$x=2:\ 1 = -3A \quad \boxed{A = -1/3}$

$= \boxed{-\frac{1}{3}\ln|x-2| + \frac{1}{3}\ln|x-5| + C}$

Substitution

3. $\int e^x \sin(e^x)\,dx$

$= \boxed{-\cos(e^x) + C}$

$u = e^x;\ du = e^x dx$

By Parts

4. $\int x \cosh(2x)\,dx$

$u = x \qquad du = dx$

$dv = \cosh(2x);\ v = \frac{1}{2}\sinh(2x)$

$= \frac{1}{2}x\sinh(2x) - \int \frac{1}{2}\sinh(2x)\,dx$

$= \boxed{\frac{1}{2}x\sinh(2x) - \frac{1}{4}\cosh(2x) + C}$

Improper Integral

5. $\int_{-2}^{2} \frac{1}{|x|}dx = 2\cdot \int_0^2 \frac{1}{|x|}dx$ (b/c $\frac{1}{|x|}$ is an <u>even</u> function)

$= 2\lim_{t\to 0^+} \int_t^2 \frac{1}{|x|}dx$

$= 2\lim_{t\to 0^+} \int_t^2 \frac{1}{x}dx$

$= 2\lim_{t\to 0^+} \left(\ln|x|\Big|_t^2\right)$

$= 2\lim_{t\to 0^+} (\ln 2 - \underbrace{\ln t}_{t\to 0^+ \,\Rightarrow\, -\infty}) = \boxed{\infty} \Rightarrow$ <u>DIVERGENT</u>