WED 10/4/2017

Substitution

1. $$\int \frac{x}{x^2+1}\,dx = \boxed{\frac{1}{2}\ln(x^2+1)+C}$$

$u = x^2+1$
$du = 2x\,dx$

Rational function (improper)

2. $$\int \frac{x^2}{x^2+1}\,dx = \int \frac{(x^2+1)-1}{x^2+1}\,dx = \int\left(1-\frac{1}{x^2+1}\right)dx$$
$$= \boxed{x-\arctan(x)+C}$$

Improper Integral

3. $$\int_0^\infty \frac{1}{x^2+1}\,dx = \arctan x\Big|_0^\infty = \frac{\pi}{2}-0 = \boxed{\frac{\pi}{2}} \text{ (fast way)}$$

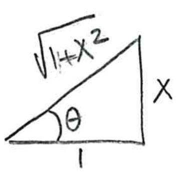
(best way)
$$= \lim_{t\to\infty}\int_0^t \frac{1}{x^2+1}\,dx = \lim_{t\to\infty}\left(\arctan x\Big|_0^t\right)$$
$$= \lim_{t\to\infty}\left(\arctan(t)-\arctan(0)\right) = \boxed{\frac{\pi}{2}}$$

Trig Substitution

4. $$\int \frac{1}{x^2\sqrt{x^2+1}}\,dx$$

$x = \tan\theta$ — $\sqrt{x^2+1} = \sec\theta$
$dx = \sec^2\theta\,d\theta$ — $\theta \in (-\pi/2, \pi/2)$

$$= \int \frac{1}{\tan^2\theta\,\sec\theta}\sec^2\theta\,d\theta = \int \frac{1}{\tan^2\theta}\sec\theta\,d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}\cdot\frac{1}{\cos\theta}\,d\theta$$
$$= \int \frac{\cos\theta}{\sin^2\theta}\,d\theta = -\frac{1}{\sin\theta}+C$$
$$= \boxed{-\frac{\sqrt{x^2+1}}{x}+C}$$



$\tan\theta = \frac{x}{1}$

$\sin\theta = \frac{x}{\sqrt{x^2+1}}$

By Parts + Partial Fractions

5. $$\int \frac{x}{(x^2+1)^2}\ln x\,dx$$

$u = \ln x$ $\qquad du = \frac{1}{x}dx$

$dv = \frac{x}{(x^2+1)^2}dx$ $\qquad v = -\frac{1}{(x^2+1)}\frac{1}{2}$

$$= -\frac{1}{2}\frac{\ln x}{(x^2+1)} - \int -\frac{1}{2}\frac{1}{(x^2+1)}\frac{1}{x}\,dx$$
$$= -\frac{1}{2}\frac{\ln x}{(x^2+1)} + \frac{1}{2}\int \frac{1}{x(x^2+1)}\,dx$$
$$= -\frac{1}{2}\frac{\ln x}{(x^2+1)} + \frac{1}{2}\int\left(\frac{1}{x}-\frac{x}{x^2+1}\right)dx$$
$$= -\frac{1}{2}\frac{\ln x}{(x^2+1)} + \frac{1}{2}\left(\ln x - \frac{1}{2}\ln(x^2+1)\right)$$
$$= \boxed{-\frac{1}{2}\frac{\ln x}{(x^2+1)} + \frac{1}{2}\ln x - \frac{1}{4}\ln(x^2+1)+C}$$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$
$1 = A(x^2+1) + x(Bx+C)$
$x=0$: $\boxed{A=1}$
$x=1$: $1 = 2+B+C$
$x=-1$: $1 = 2-(-B+C) = 2+B-$

$B+C = -1$
$B-C = -1$
$\oplus$ $2B = -2$
$\boxed{B=-1}$
$\boxed{C=0}$